



Sequential dynamics of complex networks in mind: Consciousness and creativity

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ABSTRACT

Today, based on brain imaging analyses, we can consider the brilliant metaphor about event discreteness of the conscious process by William James (1890) to be an experimental fact. Such events compose sequences: linguistic, episodic memory, motor behavior, etc., whose dynamics are robust, reproducible, and sensitively react to incoming informational signals. The human mind is able to process, understand and predict time-dependent information about the environment and about ourselves, and generate corresponding commands to control behavior. Many experiments have indicated that the mind relies on sequential dynamics to carry out these tasks. Based on brain imaging experiments, we discuss here a set of key principles and their instantiation in nonlinear differential equations to form a dynamical theory of consciousness and creativity. General hierarchical models of consciousness and creativity include coupled low-dimensional equations that govern cooperative variables for several cognitive modalities: episodic (semantic) memory, working memory, attention, emotion, perception and their sequential interaction. In the phase spaces spanned by variables of these models, the joint transient dynamics of cognitive modalities is represented by coupled heteroclinic networks which share complex *metastable states*. The interaction of such states is responsible for the robustness of transient neural dynamics involved in the generation of thoughts and in the programming of behavior. In the framework of the analyzed dynamical models, we discuss the interaction of cognitive processes and the generation of new information in creativity.

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*One of the most fascinating things
in human life is to think about
how we think, create a poem
and represent our own future.*

1. Introduction

1.1. Sequential cognitive processes

It is a pity, but we have to admit from the start that we are not planning to discuss here the question *what is the origin of the first thought?*, which resembles the question *what is the origin of life?* Do brains themselves control thoughts or is it just the action of external agents? From time to time, these fundamental questions are discussed in the literature with different provocative answers. Some authors even suppose that our Universe holds responsibility. Of course, there is no theorem proving that such hypotheses are wrong. Unfortunately, we are not ready to discuss these subjects here.

By now it seems universally accepted that human activities, including cognition, are sequences of elementary acts: operations, techniques or procedures, replacing one another in time. Just recall dancing, playing musical instruments, making coffee or speaking at a conference. Any of these processes, if abstracted from the mechanisms for generating elementary actions, can easily be modeled mathematically with the help of chain models. The meaning of the sequential items succeeding each other, of course, can be different. However, in spite of such easiness, if we think about how a program of this behavior is created in the brain, i.e., how one item of the chain is replaced by another one, and why this chain is robust, the task may seem very complicated and even unsolvable. Such impression becomes even stronger if we try to understand the future consequences of our actions in the present.

This explains the point of view that intellect and, especially, consciousness are by themselves so intriguing and even mystical phenomena that they are inaccessible to formalization and not amenable to a mathematical description. The purpose of this review is to present an alternative point of view and, if not to convince the readers, then, at least, to interest them so much that they themselves begin to reflect on how we think, and the paper will supply new ideas for this contemplation. We will focus on recent experiments, models, and evolutionary approaches to illustrate a theory, based on the methods of nonlinear dynamics, which describes the richness of cognitive phenomena: decision-making, attention, working memory, and the result of their integration, i.e., consciousness and creativity processes (see Fig. 1.1).

The success and beauty of this theory are based on the universality of the dynamical approach for the description of fundamental types of spatiotemporal activity of the human brain, including perception of the environment, thinking and decision making, action programming, and, finally, behavior. Uncovering the principles of mind dynamics is a task that relies on ideas and expertise from a number of different inter-related disciplines like neuroscience, physics, psychiatry, linguistics, philosophy, social sciences, neuro-technology and others. Universality of these principles may also imply a new approach for the creation and analysis of artificial intelligence systems and biomedical applications as we will also discuss below.

From this perspective, “the mind” can be seen as a functional space that contains mathematical images of different cognitive processes. The focus on robust sequences to describe such processes allows an adjustable low-dimensionality for this space. If the space harbors many modalities of thought, it should be endowed with a hierarchical structure and mechanisms for the coordination of dynamics — synchronization, binding, and chunking [1].

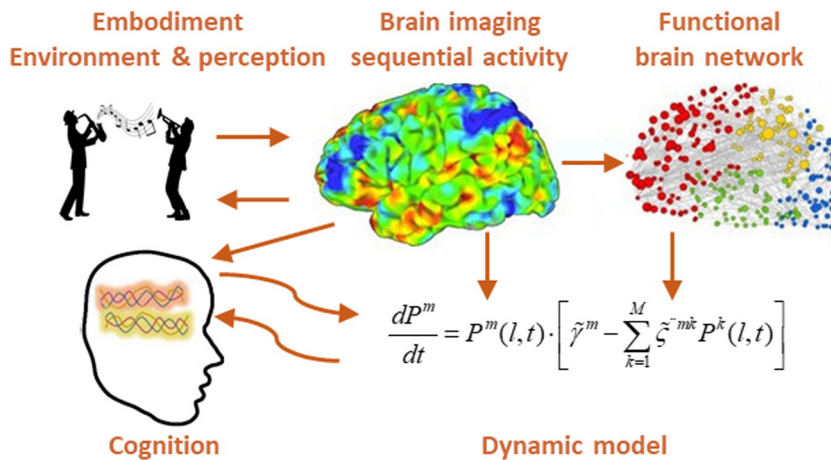


Fig. 1.1. Nonlinear dynamics provides a key to link brain and cognitive activity on different levels of the neural system's hierarchy. Brain imaging experiments are used to describe the dynamics and structure of functional neural networks. Dynamical models integrate this information to disclose fundamental principles observed in imaging and electrophysiological experiments, such as the robustness of sequential processing in cognitive activity, e.g., in creative tasks such as jazz improvisation. Predictions from these models can be used in new interpretations of experimental observations and in biomedical applications.

1.2. Sequences and metastable states

The main current interest in cognition is focused on the understanding of the relation between the architecture (connectome) of the global brain network [2] and the dynamics of the corresponding mental processes [3–7]. To address this goal we need to formulate mathematical models based on dynamical principles that support experimentally observed features of hierarchical sequential processes [8–12] (see also an alternative point of view in [13]), such as stability against spatiotemporal noise on different scales, reproducibility, and minimization of the resources.

There are two different but interdependent dynamical hierarchical structures that are related to consciousness. The first one dynamically links stages of brain organization from genes to consciousness [14]. The second hierarchy is the organization of the different mental processes like attention, working memory, binding, chunking, etc., in the mental space or in the phase space of corresponding dynamical models [15].

Many experiments suggest that the brain uses discrete economic strategies to represent, generate and process cognitive information, e.g. [9,16–18]. These strategies include the analyses of hierarchical recurrent brain networks that enable and provide low-dimensional descriptions. In particular, such analyses can address self-awareness dynamics that is responsible for interacting with the environment [19,20]. Sequential transient brain dynamics can be seen as the result of sequential switchings of observable metastable states [21–23]. Formally, “meta-stable” means “beyond stability”, but not necessarily “beyond existence”. This includes situations with invariant sets still present but unstable. The system approaches such states (unstable equilibria or unstable periodic orbits) along their stable manifolds, hovers for a certain time sufficiently close to them and departs along the unstable manifolds [24]. In the theory of dynamical systems, the paths (phase trajectories) between different metastable states are called “heteroclinic trajectories”.

We will see below that many patterns of cognitive activity can be considered as built around such heteroclinic structures. The corresponding variables that organize this space depend on the type of measurement. One of the most popular and effective approaches to select such variables is encoding of mental processes by means of principal component analysis [25] that within the last three decades has been successfully applied to brain data [26,27] (see Fig. 1.2).

At first glance, the world seems to us a continuous stream of percepts. However, recent experiments suggest another point of view in which the integrated processes of perception could operate in a discrete manner, just like movies consist of discrete scenes [28].

1.3. Winnerless competition principle

In the last decade, under the influence of impressive experimental work, studies of the spatial and temporal activity of the brain during the performance of cognitive functions (analysis of functional magnetic resonance patterns) have established that the functional cognitive activity of the brain is a robust transient process [29,30]. In addition, evidence suggests that consciousness and creativity result from mutually coordinated activity of several cognitive subnetworks in the brain [31]. Different clusters of networks perform different cognitive functions. Each of these networks unites a large number of brain substructures. Transient dynamics, corresponding to the cognitive process, is not a homogeneous drift across the mind; rather, the mind sequentially wanders from one item to another. As a consequence of mutual inhibition

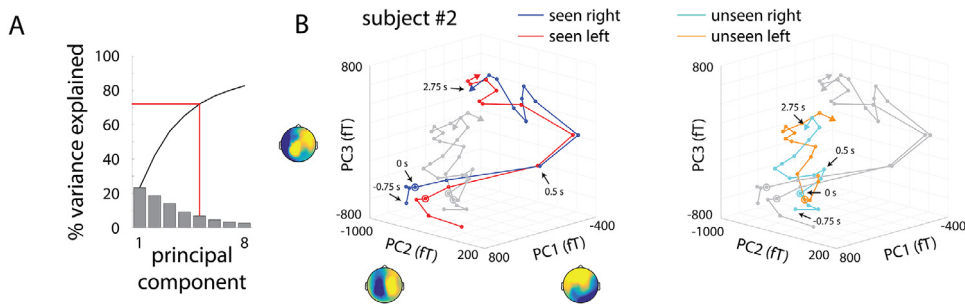


Fig. 1.2. Encoding conscious visual perception in robust, transient mental dynamics. The plots correspond to the processing of magnetoencephalographic (MEG) data with high resolution in space and time and the associated principal component (PC) analysis. (A) Distribution of variance over principal components. (B) Trial-averaged activity trajectories in the subspace, spanned over three leading principal components for seen and unseen trials separately, under the presentation of the right-tilt (blue and cyan) or left-tilt stimulus (red and orange). Both plots show the same trajectories (i.e. gray trajectories in each plot are the same as colored trajectories in the other plot), units are in femtotesla (fT).

Source: Adapted with permission from [27].

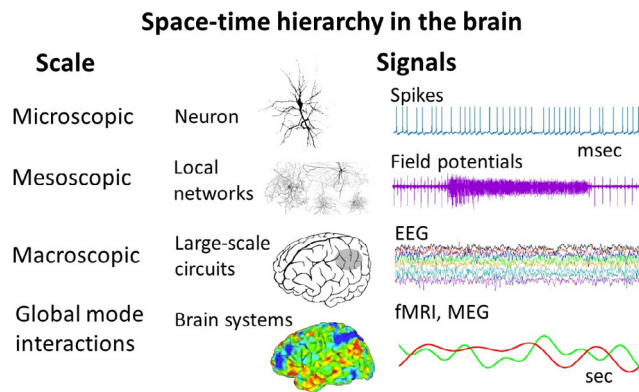


Fig. 1.3. Schematic representation of the space–time hierarchy in the brain, as measured by different experimental techniques. Intracellular and extracellular recordings measure neuron activity at the single neuron and local network levels. Electroencephalography (EEG), functional magnetic resonance imaging (fMRI) and magnetoencephalography (MEG) can record sequential activity at different brain regions with complementary temporal and spatial resolution.

among the items, the process cannot find ultimate rest near one of them: the new local mode of instability drags it further to the next item. Taken together, this combination of instabilities results in the perpetual successive change of the items, governed by the WinnerLess Competition principle (WLC) [32].

The WLC is observed in nonlinear dissipative multi-agent systems of different nature: ecological, social, physiological etc. In brain dynamics, the WLC principle is closely related to another fundamental principle of cognition: the low-dimensionality of functional cognitive dynamics [15,21,33–35].

By cognitive dynamics we refer to the dynamical aspects of cognitive processes. In our analysis, we do not focus on the details of the physical brain elements that support cognitive activity. From this perspective, we describe the processes of thinking and creativity, which require the integration of distinct spatiotemporal scales (see Fig. 1.3) of different cognitive modalities. Thus, we employ methods and results of quantitative studies of mental activity and its features as a dynamical process. The dynamical patterns that underlie robust sequential dynamics are associated with information flows whose main features can be described by a heteroclinic network model approach.

The corresponding dynamical theory is based on finding the relationships between the brain global networks (as observed, for example, in *functional magnetic resonance imaging* (fMRI) experiments) and the dynamic brain patterns (visualized in the form of different phase portraits). Since the neuronal activation of specific brain regions is accompanied by the increase of cerebral blood flow through them, this technique delivers adequate maps of local and global brain activity, see an example in Fig. 1.4.

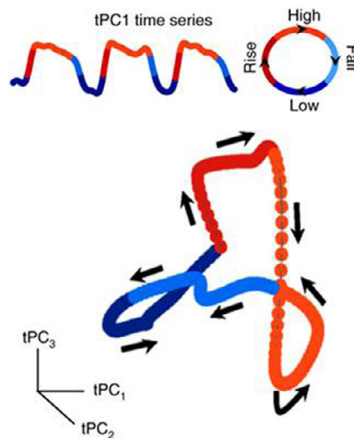


Fig. 1.4. Low-dimensional signatures of cognitive tasks. The time series of each principal component (tPC) of fMRI recordings is used for providing a low-dimensional subspace in which the state space manifold is embedded. The low-dimensional manifold is traversed by the global brain state across the first three dimensions, with arrows depicting the direction of flow along the manifold.

Source: Adapted from [36] with the permission of Springer Nature.

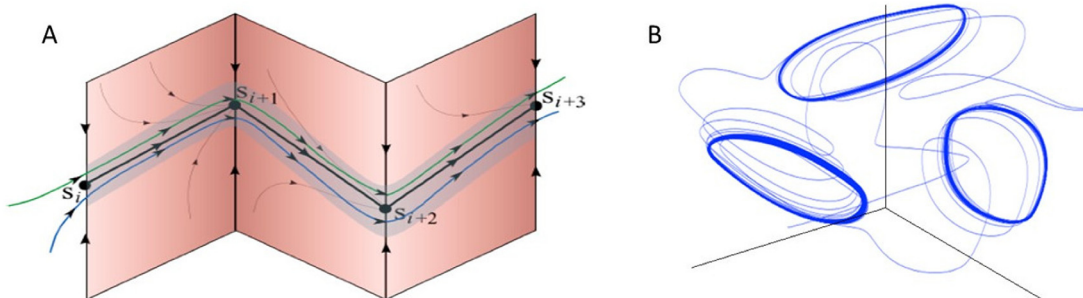


Fig. 2.1. Illustration of a heteroclinic channel, a useful mathematical object enabling robust transient dynamics, in the phase space. Panel A: sequence of static metastable states representing the informational items: the saddle equilibria S_k , with index k numbering the items. Panel B: sequence of dynamical metastable states — in this case, the mathematical images of these informational items are saddle periodic orbits.

Source: Adapted from [43].

2. Functional cognitive heteroclinic networks

2.1. Elements of heteroclinic networks in the phase space

Spatiotemporal functional network dynamics can be considered as sequential [37–39]. The vast variety of cognition and behavior can be better represented when understood through the temporal switching between different network modes (*cognitive modes*). To be robust and computationally efficient, the dynamics of these modes has to satisfy a set of principles that we will discuss below.

Cognition is a transient process, it is neither a state of equilibrium nor an exactly periodic oscillation, therefore in the corresponding phase space it cannot be effectively represented by attractors with predictable dynamics, such as stable fixed points, limit cycles or tori. For the brain to effectively adapt to handle multiple flexible cognitive processes, it must be able to work in transient modes for a specific cognitive task [22,40]. A convenient approach is to represent these processes as metastable states that emerge in a spatiotemporal setting [41,42]. In this way, a number of cognitive processes can operate with similar modes, e.g. over the similar set of the spatial patterns, and can achieve numerous goals through a wide variety of switching sequential patterns across different metastable states [21,34] see Fig. 2.1. In this context, metastable states represent informational items related to cognitive coding, such as a sensory perception (e.g. visual, auditory, olfactory patterns), a cognitive decision, a memory engram, etc. Some items can require a single metastable state while others need a chain of such states.

Cognitive/mind dynamics that corresponds to global brain mode competition satisfies the Winnerless Competition principle (WLC). The WLC principle was first formulated to describe spatiotemporal coding in sensory information [32,34,44]. Sequential coding of sensory information means including time in the coding space (see Figs. 2.2 and 2.3).

Beyond sensory processing, the WLC principle can also be directly related to the general sequential dynamics of metastable brain states that are activated by internal or environmental stimuli. WLC dynamics is typical for brain

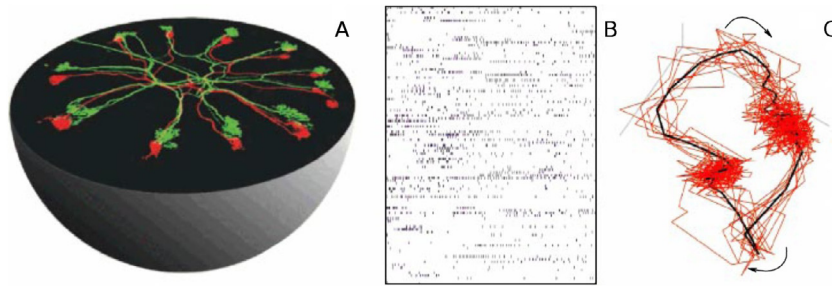


Fig. 2.2. Experimental evidence for transient sequential dynamics in the encoding of the insect olfactory system. The panels illustrate the spatio-temporal representation of sensory information in the locust antennal lobe [36]: (A) Schematic representation of the antennal lobe sectioned through its equatorial plane; (B) response of 110 antennal lobe neurons to an odorant lasting 1.5s; (C) projection of neural activity on a 3D principal component space (black trajectory is the average of 10 different experiments).

Source: Adapted with AAAS permission from [34].

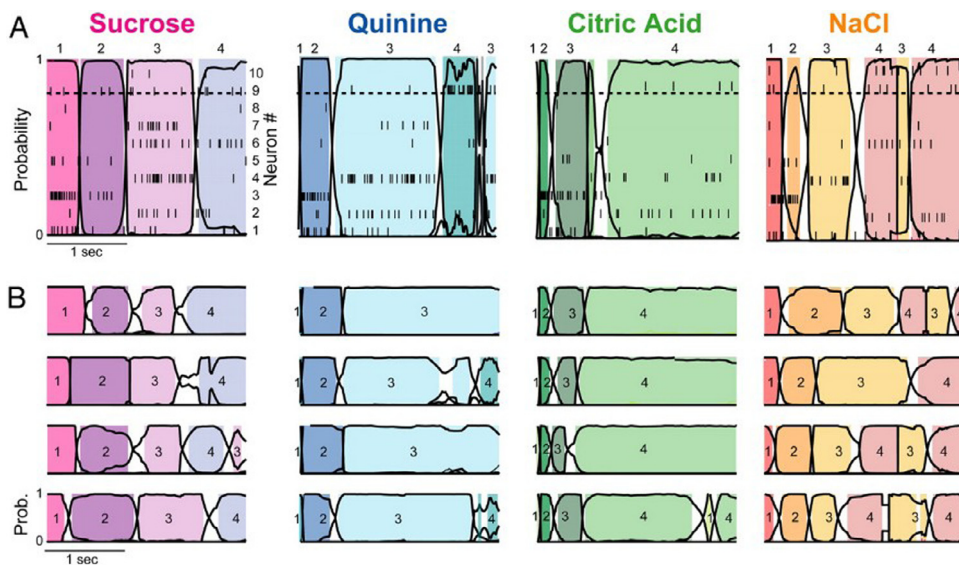


Fig. 2.3. Experimental evidence for transient sequential dynamics in mammals. Rat gustatory cortex neurons generate taste-specific sequential patterns [45]. (A) Panels show the sequential activations among 10 cortex neurons in response to four taste stimuli: sucrose, quinine, citric acid and NaCl. (B) Four additional sequences for the same neuron ensemble showing the reliability of the sequences. Sequences are highly reproducible in spite of the irregularity in their switching times.

Source: Adapted with permission from [45]. Copyright 2007 National Academy of Sciences.

functional networks with excitatory and inhibitory connections [46]. This approach allows us to adequately describe a wide variety of cognitive phenomena such as the limitations of working memory capacity [47–49], attention focusing and attention switching [43,50,51], the dynamical interaction between emotion and cognition [52], speech production [53] and, potentially, the dynamics of mental disorders [52,54].

2.2. Robustness against noise and sensitivity to informational signals

Since cognitive dynamics, as a rule, involves many metastable states, a new global dynamical object is required to describe sequential activity that does not depend sensitively on initial conditions: this is a stable heteroclinic channel (SHC) — a narrow pathway in the phase space, built around the appropriate itinerary of heteroclinic orbits. Depending on the particular situations, the channel can be closed (and then the dynamics becomes cyclic) or open. Open heteroclinic channels serve as the pathways of transient sequential activity in neural circuits [55]; the conditions in which such channels are robust (i.e. the trajectories do not leave them in the middle of the path) are formulated in terms of the linearization eigenvalues of the participating metastable states of equilibrium.

Despite of their relative robustness against noise, cognitive tasks are sensitive to small environmental and intrinsic stimuli. There is a growing recognition that the adaptive networks which support cognition change over time, and that some aspects of these connections may be transient [56]. These can also be seen as an interconnected network formed by

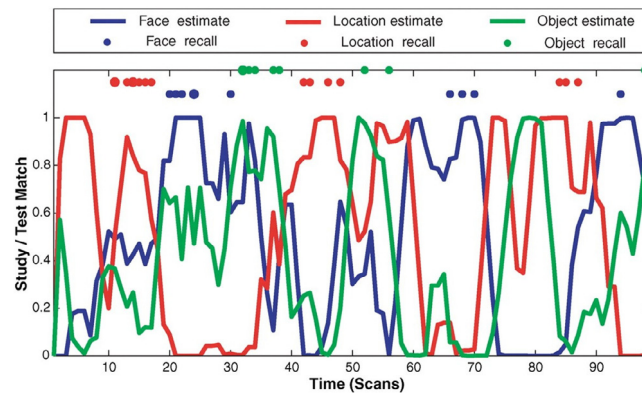


Fig. 2.4. Illustration of how sequential brain activity during a memory recall relates to recall behavior. Each point on the x-axis corresponds to a 1.8 s interval (during the 3-min recall period). The blue, red, and green lines correspond to a classifier's estimate as to how strongly the subject is reinstating cognitive sequential patterns characteristic of face-study, location-study, and object-study at that point in time. Each line indicates the primacy of a given network in the sequential switching between cognitive processes overtime. The graph illustrates the strong correspondence between the classifier's estimate of category-specific brain activity, and the subject's actual recall behavior.
Source: From [69]. Reprinted with permission from AAAS, cf. Fig. 1.1.

active dynamic states with hierarchical connective properties and by other nodes that carry out more sensory or reactive roles [11] (see below). A possible resolution to the fundamental contradiction between robustness and sensitivity in WLC dynamics arises from the fact that noise and informational signals influence the SHC in two qualitatively different ways. Noise can just change initial conditions for the trajectories inside the channel, and if inhibition is strong enough, the disturbed trajectories stay captives of the channel, whereas the channel itself is robust. In contrast, small information signals can excite new participants: modes that qualitatively change the architecture of the SHC. Such alteration of the channel topology represents the reaction to new information.

Recently, many investigators have suggested the inclusion of a temporal dimension when encoding cognition. For example, Janoos et al. investigated the temporal structure of spatial activity maps in a state-space model for cognitive functions which provides a spatially varying estimate of the hemodynamic response [57]. Similarly, Kriegeskorte et al. focused on the low-dimensional feature-space for representing the data [58]. In this last paper, the set of fMRI data acquired during a mental arithmetic task was analyzed to characterize the spatiotemporal information about specific mental processes. Research in language comprehension and decoding provides a clear example of the importance of including time in the cognitive coding space, requiring the dynamic interaction between multiple brain regions [59]. Furthermore, recent EEG-based experiments have disclosed the existence of *microstates* — short-lived basic patterns of electric fields [60] that can be associated with metastable states. A general approach to model microstates, that aims at capturing the statistical properties of microstate sequences has been proposed in [61].

2.3. Sequential cognitive activity

Neuroimaging research over the past decade has revealed a detailed picture of the intrinsic organization of the human brain. However, because of the complexity of neural organization and the huge variability of cognitive functions, finding an exact mapping between brain activity and behavior is an extraordinarily difficult problem, with few exceptions (e.g. see Fig. 2.4).

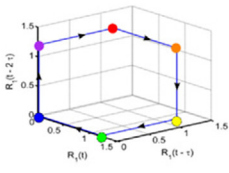
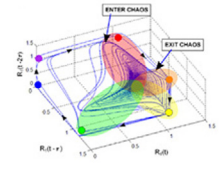
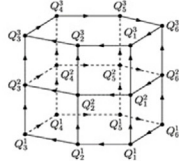
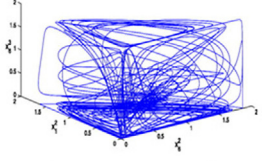
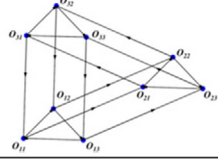
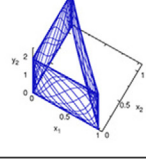
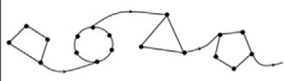
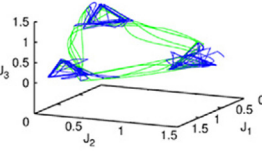
Brain imaging methodology has been primarily data-bound, typically imaging data is collected in 3-dimensional pixels (voxels) over a time dimension that yields a 4-dimensional matrix. The time series for these voxels are associated with either an external stimulus (using a correlation approach) [62–64] or an internal stimulus (using a connectivity approach) [65,66]. These approaches attempt to determine meaningful data-derived signals. Multi-voxel pattern analysis [67] and multivariate Bayesian decoding [68] provide a way to integrate multiple voxels in predicting or classifying brain states. However, with our understanding of the existent parameters of human cognition, we propose an analysis that is based on a relatively low-dimensional predictive non-linear dynamic modeling; the models can be fitted to existing data and extended towards extrapolation of data. The specific features of functional cognitive dynamics, i.e., phase portraits and time series, are determined by the architecture of the corresponding large scale brain networks (see Table 1). Such modeling approach allows designing heteroclinic skeleton architectures to naturally implement multiscale coordination phenomena such as synchronization, binding or chunking (grouping).

2.4. From main principles to heteroclinic dynamics

Derivation of a generalized dynamic model of cognitive processing requires the formulation of a set of mathematical equations that dictate adherence to generalizable principles [50]. Thus, the approach requires that: (i) equations govern

Table 1

Heteroclinic skeleton networks, phase portraits in their neighborhood and associated cognitive phenomena represented by these dynamical objects.

HETEROCLINIC NETWORKS AND EXAMPLES OF ASSOCIATED COGNITIVE PHENOMENA		
Heteroclinic skeleton networks	Phase portraits of trajectories close to the heteroclinic skeleton	Examples of associated cognitive phenomena and references
		Heteroclinic chimera in a stable heteroclinic network with several metastable states Rabinovich et al. Frontiers in Systems Neuroscience 8: 220 (2014).
		Binding heteroclinic network Afraimovich et al., Chaos, 25: 103118 (2015) Rabinovich & Varona, Front. Comput. Neurosci. 12: 73 (2018).
		Heteroclinic synchronization and heteroclinic tori Afraimovich et al., Chaos, 28 053107 (2018).
		Chunking of components with different time scale Rabinovich et al, Front. Comput. Neurosci. 8 (2014).

the variables that represent the evolution of neural elements in their temporal coherency, and solutions of those equations correspond to metastable patterns associated to cognitive processes; (ii) the model is endowed with WLC dynamics – a nonlinear process of interaction among many informational items in the form of spatiotemporal modes – that enables sequential switching between metastable states and the potential robustness of transient creativity dynamics, (iii) the model exists as an open dissipative system with overall balance between inhibition and excitation, and (iv) the dynamics of the model has to be sensitive with respect to memory and environment information.

Projection functions model the spatial components of the modes and can be interpreted as signatures of the functional brain networks on which cognitive processing is based. We represent the spatial and temporal dependence of such processes as a superposition:

$$R(l, t) = \sum_{m=1}^M P^m(l, t) \quad (2.1)$$

where $P^m(l, t) = R^m(t)Q^m(l)$ denotes the m th spatio-temporal mode that depends on time t and is based on the set of discrete coordinates l in the physical brain space, the non-negative real function $R^m(t)$ describes the temporal evolution of the m th mode whose spatial structure is represented by $Q^m(l)$, the projection function of the m th mode, and M is the overall number of modes. We are seeking a *normal form* – a structure of the minimal complexity (e.g. a polynomial of the lowest degree) that ensures all relevant types of dynamics. To keep the amplitudes non-negative, the right hand side of the evolution equation for each mode should be proportional to the amplitude of this very mode. Accordingly, the normal

form in this situation is, for each mode, an ordinary differential equation with the simplest quadratic nonlinearity:

$$\frac{dP^m}{dt} = P^m(l, t) \cdot \left[\tilde{\gamma}^m - \sum_{k=1}^M \tilde{\zeta}^{mk} P^k(l, t) \right]. \quad (2.2)$$

Here, $\tilde{\gamma}^m$ is the level of excitation of the mode m , whereas $\tilde{\zeta}^{mk}$ is the level of its inhibition by the mode k . Substituting here (2.1) with subsequent summation, we arrive at the form of the canonical ecological model – the set of generalized Lotka–Volterra (GLV) equations [43]:

$$\theta_m \frac{dR^m}{dt} = R^m \left[\tilde{\gamma}^m(X^m, S^m) - \sum_{k=1}^M \tilde{\zeta}^{mk}(R^k, S^m) R^k(l, t) \right]. \quad (2.3)$$

where X^m is the input from the other levels of the network hierarchy, as defined below in Eq. (2.4), whereas S^m is the sensory input for the m th modality. Here, $\theta_m = \sum_l Q^m(l) \geq 0$, $\gamma^m = \tilde{\gamma}^m \theta_m$ and the matrix elements $\zeta^{mk} = \tilde{\zeta}^{mk} \sum_l Q^m(l)$ $Q^k(l)$ are non-negative. The interaction among K^m cognitive modalities, e.g., attention, perception and emotion, can be represented by their own sets of modes (cf. (2.1)):

$$X^m(q, t) = \sum_{i=1}^{K^m} x_i^m \phi_i^m(q^m) \quad (2.4)$$

where ϕ_i^m is the projection function of the i th mode of the m th modality. We arrive at the generalized canonical form:

$$\tau_i^m \frac{dx_i^m}{dt} = x_i^m \cdot \left[\sigma_i^m(R^m, S^m, C^m) - \sum_{j=1}^{K^m} \rho_{ij}^m x_j^m - \sum_{k=1}^M \sum_{j=1}^{K^m} \xi_{ij}^{mk} x_j^k \right] \quad (2.5)$$

$$\tau_i^m = \sum_{q^m} \phi_i^m(q^m), \quad (2.6)$$

where ρ_{ij}^m is the inhibitory connectivity matrix between modes of the same modality, and ξ_{ij}^{mk} is the connectivity matrix between modes of different modalities. It is reasonable to assume that $\sigma_i^m \sim \tau_i^m$, $\rho_{ij}^m \sim \sum_{q^m} \Phi_i^m(q^m) \Phi_j^m(q^m)$, $\sigma, \rho \geq 0$.

Remarkably, the generalized Lotka–Volterra equations also naturally appear in the continuum limits of large-scale neural networks, i.e., in neural field models, where they characterize stability of spatially non-constant stationary states in heterogeneous fields for specific synaptic interaction kernels [70]. Lotka–Volterra models are a convenient framework to discuss dynamical aspects of synchronization, coordination and binding [44,71–75]. An important property of the generalized Lotka–Volterra equations is the presence of many invariant hyperplanes in their phase space: if any of the variables R^m in (2.3) or x_i^m in (2.5) is initially set at zero, it stays zero all time. A corollary of this, most important in our context, is the robust heteroclinic dynamics. In generic dynamical systems, the heteroclinic connections (trajectories) between the metastable states of equilibrium are structurally unstable (see below), and exist only at specific combinations of the governing parameters: a minor variation of the parameters destroys them. In contrast, within the class of generalized Lotka–Volterra equations, the heteroclinic trajectories between the saddle points are structurally stable and exist in open regions of the parameter space.

2.5. Existence and stability of heteroclinic sequential dynamics. Bifurcations. Fluctuations.

In this subsection, we discuss the robustness and stability of heteroclinic trajectories between the metastable states. We call the heteroclinic cycle or channel *stable* if all trajectories that start in its sufficiently close vicinity, stay forever in the neighborhood of this cycle/channel. Concerning *robustness*, we denote by this term the *structural* stability of the vector field: the heteroclinic connection is robust, if it persists at all sufficiently small variations of the system parameters.

Here, for brevity, we restrict ourselves to metastable states that are just hyperbolic saddle equilibria $S_1, \dots, S_N$ of an n -dimensional dynamical system [76] (see panel A of Fig. 2.1). Heteroclinic connections between the saddles in dissipative dynamical systems are generically fragile due to the topological argument. A trajectory from S_1 to S_2 should simultaneously belong to the unstable manifold W_1^u of S_1 and to the stable manifold W_2^s of S_2 . If the dimensions of stable manifolds coincide: $\dim(W_2^s) = \dim(W_1^s)$, then W_1^u and W_2^s generically would not intersect along the (at least) one-dimensional curve; this hinders the existence of the robust heteroclinic trajectory. If $\dim(W_2^s) > \dim(W_1^s)$, there is a generic connection from S_1 to S_2 , but no generic connection from S_2 back to S_1 . A longer chain of robust connections would imply the growth of the dimension of the stable manifold along the chain. For obvious reasons, this is possible only for finite (commonly, quite short) lengths, and forbids the existence of closed contours.

To enable robust heteroclinic connections between the saddles, a dynamical system should be endowed with a particular structure of the phase space. Two classes of such systems that often appear in applications are (i) the systems with discrete permutational symmetries [77] and (ii) the systems whose phase space is sliced by a sufficiently large set

of invariant (hyper)planes [78]. The setup in which the variables can be permuted includes the very first documented example of robust heteroclinic contour between three unstable patterns: the description of thermal convection in a rotating fluid layer [79], as well as different modifications of systems of globally coupled identical one-dimensional phase oscillators where the structurally stable heteroclinic orbits result in the phenomenon of so-called “slow switching” [80–83]. Robust heteroclinic connections can also arise in the situations where the individual networks are identical but the coupling pattern is heterogeneous [84]. Permutations and/or units with identical on-site dynamics seem to be hardly relevant for our context: different modalities in the brain refer to fundamentally different activities and, hence, are not interrelated by permutation symmetries. Instead, in cognitive dynamics the situation (ii) is widespread: as recognizable in the canonical equations (2.2) or (2.5), the generalized Lotka–Volterra models offer a rich structure of invariant hyperplanes upon which groups of the variables identically vanish. Here, in a typical situation, the heteroclinic trajectory between two saddle steady states entirely lies inside the quadrant of the two-dimensional invariant plane; within this plane one steady point is a saddle whereas the other steady point is the sink. Since the reduction of the Lotka–Volterra equations with inhibitory quadratic nonlinearities onto this quadrant can contain neither orbits going to infinity nor closed phase trajectories, the unstable manifold of the first point is obliged to end in the sink, thus serving for the robust (as long as the canonical setup is preserved) heteroclinic connection.

Existence of heteroclinic orbits is not sufficient for their observability: for this purpose, stability with respect to sufficiently small perturbations is required, so that the trajectories that start close to heteroclinic solutions, approach them in the course of time. For the trajectories in the heteroclinic channel, the overwhelming proportion of time is spent in slow motion near the steady states, therefore their stability is dominated by the properties of linearization near those states. Let the eigenvalues $\lambda_1^{(i)}, \dots, \lambda_n^{(i)}$ of the Jacobian matrix of the system linearized at the i th equilibrium S_i be ordered in such a way that $\lambda_1^{(i)} > \dots > \operatorname{Re} \lambda_m^{(i)} > 0 > \operatorname{Re} \lambda_{m+1}^{(i)} \geq \dots \geq \operatorname{Re} \lambda_n^{(i)}$. Then, on the m_i -dimensional unstable manifold W_i^u of S_i there is a strongly unstable one-dimensional manifold, tangent to the first eigenvector. The ratio of eigenvalues

$$v_i = -\frac{\operatorname{Re} \lambda_{m_i+1}^{(i)}}{\lambda_1^{(i)}} \quad (2.7)$$

is known as the saddle value [85]. Heteroclinic channels, or cycles, are attracting provided that the product of all saddle values in the channel $P = \prod_i v_i$ is larger than 1. Geometrically, the condition $P > 1$ implies that during the passage along such channel, contraction in the phase flow dominates over expansion. Notably, each saddle, taken alone, is not obliged to keep its saddle value above 1: what matters for the stability of the motion along the channel is the product over all metastable steady states in the channel. In this way the strongly contracting saddles can counteract the destabilizing action of the weakly expanding ones.

Within the class of the generalized Lotka–Volterra equations (2.5), the robust heteroclinic orbits exist in large regions of the parameter space, and are stable in the sizeable subset of the domain of their existence. Statistics for the probability to observe sequential heteroclinic dynamics in a set of canonical equations with randomly chosen coefficients was computed and discussed in [86]. Variation of parameters may lead the system outside the stability region: destabilize or destroy the heteroclinic channel. For general analysis of this situation see e.g. [87]. Among the possible mechanisms of destruction, relevant for our setup, we mention the situation where the new state of equilibrium enters the positive orthant of the phase space. On the way, it collides and exchanges stability with one of the saddle points participating in the heteroclinic connection. As a result, the heteroclinic orbit disappears in the course of this *transcritical heteroclinic bifurcation* [88]; it is replaced by the stable state of equilibrium.

Another bifurcation scenario is related to the changes in the local characteristics of the saddle points. If, in the course of the parameter variation, the product of all saddle values (2.7) turns smaller than 1, the heteroclinic cycle, albeit still existing, can lose asymptotic stability [89]. A usual stable limit cycle with long (but finite!) period bifurcates from the heteroclinic trajectory and inherits its stability; locally, the sequential character of behavior does not change much: the attracting trajectory is still passing at close (but finite!) distances from the metastable states.

In the simplest configuration, unstable manifolds of all saddles are one-dimensional. An increase in the dimension of unstable manifolds enriches the dynamics, creating possibilities for scenarios that are more elaborate than the mere cyclic repetition (see e.g. [90]). In the situation when the unstable manifolds of the equilibria are two-dimensional, their union can form an attracting non-smooth two-dimensional “heteroclinic torus”. Although on its invariant surface no chaos is possible, every single orbit is unstable [91]. Another kind of two-dimensional attractor for the sequential dynamics, equivalent to the flow either on the 2d-cylinder or on the Möbius strip, has been described in [92]. The hierarchical setup of heteroclinic orbits, characterized by the interaction of several typical timescales, was addressed in [93]. An interesting variant for the solution of the, in a sense, *inverse problem*, was suggested in [94]: if a heteroclinic network with known properties, based on the set of saddle equilibria, is desired, what should be the connectivity matrix? Algorithms for the identification of metastable states and heteroclinic connections in experimental data, including the recordings of event-related brain potentials, have been proposed in [95,96]. Furthermore, cognitive sequential activity can be viewed in the general context of *chaotic itinerancy*, see e.g. [97,98].

Dynamical systems (2.2) and (2.3) are deterministic. On the other side, neuronal dynamics at the microscopic level is permanently affected by various kinds of fluctuations. The most straightforward effect of additive noise was reported already in the very first publication on heteroclinic cycles by Busse and Heikes in 1980: “existence of noise prevents the

amplitudes from decaying to arbitrary small levels” [79]. As a result, the trajectories are forced to leave the vicinity of metastable states, and the duration of the motion effectively becomes finite: the heteroclinic orbit is replaced by a noisy limit cycle. For estimates of the dependence of duration on the noise amplitude, see e.g. the paper of Kifer from 1981 [99] and later work [100]. A richer, highly non-trivial role is played by noise when it acts upon not a single heteroclinic cycle, but a heteroclinic network: a union of several heteroclinic cycles sharing some of the equilibria and connections between them. A competition between the different heteroclinic cycles in the network is influenced by noise [101]: depending on the character of the underlying deterministic dynamics, trajectories may display random switching between the heteroclinic cycles, or, in contrast, follow some of those cycles within arbitrarily long time, reinforcing or reversing the deterministic preferences. For the heteroclinic orbits that arise in the generalized Lotka–Volterra equations (2.5), it is reasonable to distinguish between the action of the additive noise (here, special precautions should be met to ensure that all variables stay positive), and the multiplicative noise: the relative contribution of the latter wanes when the system approaches the states of equilibrium. Analysis, performed in [102], showed that properly tuned additive noise is able to replace the heteroclinic cycle by the limit cycle in its vicinity; this would result in a narrow-band rhythmic activity of the system. Unlike additive noise, the multiplicative one does not destroy the heteroclinic cycle: duration of stay near metastable states grows when the intensity of the multiplicative noise is raised, and beyond a threshold noise level the sequential switching ceases. In the more general context this implies that heteroclinic channels are persistent against multiplicative noise, and are able to transmit information with high signal-to-noise ratio.

2.6. Complex dynamical networks in simple systems

The hierarchical canonical model described above is based on a neuronal activity rate description and can be generalized for the case of spiking neuronal ensembles, see [103]. Presence of non-symmetrical reciprocal inhibitory connections between neuronal groups that form specific cognitive modes requires *stable heteroclinic channels*. Both rate- and spiking-based canonical models have SHCs in their phase spaces. In such models, SHC implements robust sequential dynamics, which is typical for a wide variety of cognitive activities. Heteroclinic dynamics delivers an appropriate mathematical framework for transient processes that can be treated as an itinerary pass through metastable states, see [21,104].

The complexity and diversity of such processes, i.e., the complexity of the architecture of the heteroclinic network in the cognitive phase space, depends on two factors: (i) the number of degrees of freedom, i.e., the number of variables that form a model, and (ii) the number of metastable states with several unstable directions that coexist in a fixed region of the control parameter space. This last feature depends on the kind of nonlinearity in the canonical model.

For incorporating a huge number of metastable states, the dynamical system does not need to be of very high order. In several recent publications, four-dimensional hyperchaotic systems with versatile complex dynamical behavior have been proposed [105,106]. In particular, in these papers a formal method has been suggested for constructing in a low-dimensional phase space an infinite number of attractors that can be connected by separatrices (see Fig. 2.5). We illustrate the transformation of a unique compact attractor into a multi-scroll set, extended along several directions, by an example from [106] where the starting point is a system of 4th order, governed by the equations

$$\begin{aligned}\dot{x}_1 &= ax_2 - ax_1 - ex_4 \\ \dot{x}_2 &= bx_1 - x_2 - x_1x_3 - fx_4 \\ \dot{x}_3 &= x_1x_2 - cx_3 \\ \dot{x}_4 &= kx_2x_3 - dx_4\end{aligned}\tag{2.8}$$

At appropriate values of the parameters the attractor of the system is chaotic. The coordinate transformation $x_1 = R \sin y_1$, $x_2 = R \sin y_2$, $x_3 = R \sin y_3$, $x_4 = y_4$ leads to the set of equations

$$\begin{aligned}\dot{y}_1 &= \frac{aR(\sin y_2 - \sin y_1) - ey_4}{R \cos y_1} \\ \dot{y}_2 &= \frac{R(b \sin y_1 - \sin y_2) - R^2 \sin y_1 \sin y_3 - fy_4}{R \cos y_2} \\ \dot{y}_3 &= \frac{R \sin y_1 \sin y_2 - c \sin y_3}{\cos y_3} \\ \dot{y}_4 &= R^2 k \sin y_2 \sin y_3 - dy_4\end{aligned}\tag{2.9}$$

Since the transformed equations are invariant with respect to translations of y_1, y_2, y_3 by multiples of 2π , the compact chaotic attractor of (2.8) turns, in terms of the variables of (2.9), into a multiscroll pattern. The phase portrait of the system (2.9) extends into the directions y_1, y_2, y_3 ; the process of traveling along these involved spatial structures reminds Lévy flight dynamics [107].

Following this logic, below we use two types of models: generalized Lotka–Volterra equations with minimal (quadratic) nonlinearities, and the complex Ginzburg–Landau model that is characterized by a cubic nonlinearity. This approach is especially useful for the design of nonlinear dynamical models that include both power (amplitude) and phase variables.

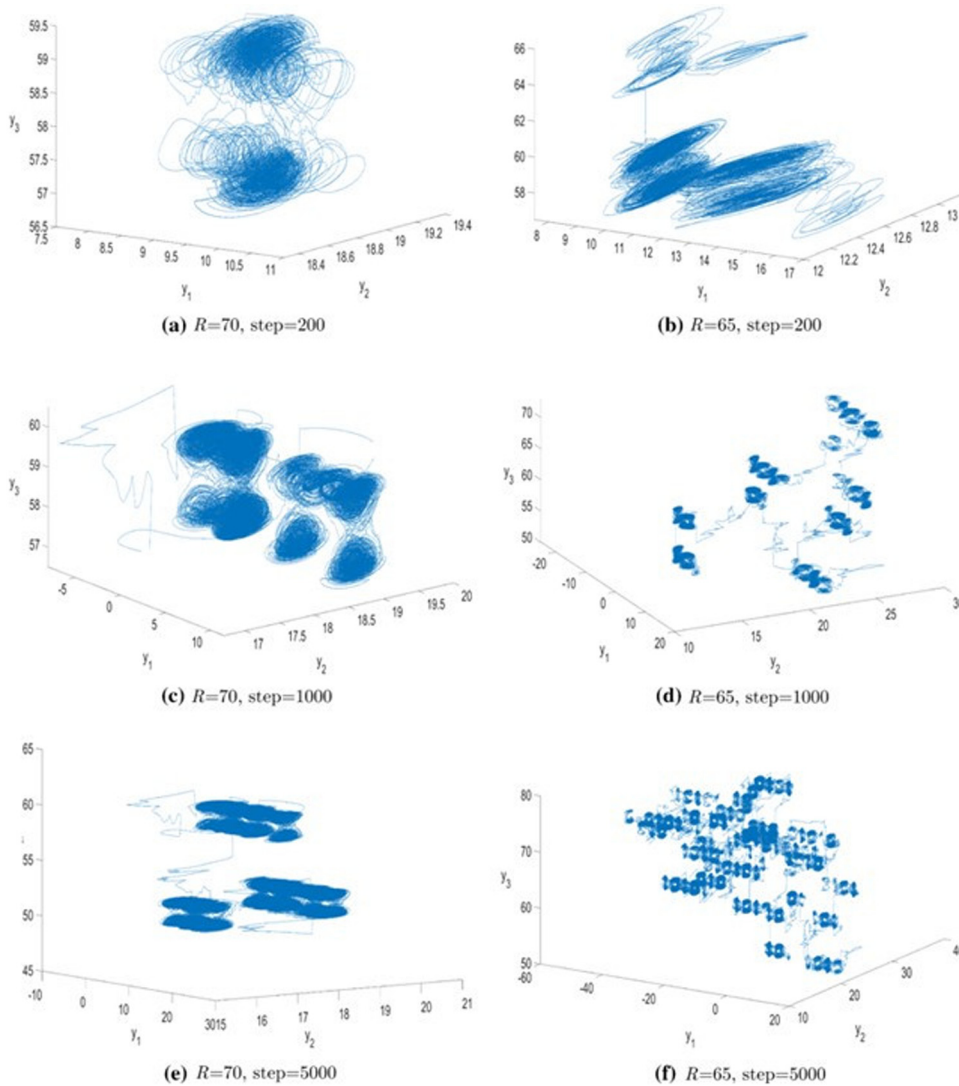


Fig. 2.5. Multi-directionally extended multi-scroll attractors of the system (2.9) at $a = 7$, $b = 50$, $c = 3$, $d = 10$, $e = 5$, $f = 5$, $k = 1.5$.
Source: Adapted from [106] with the permission of Springer Nature.

2.7. Limited information capacity and stability

There is a limitation in the processing of large simultaneous amounts of information streams that are received both from external stimuli and from the brain internal dynamics. The process of attention sequentially selects the most important information for a specific cognitive task. The associated cognitive performance depends on the interaction between short-term working memory (WM) and the attentional processes [108].

The capacity of sequential working memory has been estimated to be in the range of 5 ± 2 items [109]. From the dynamical point of view, this limitation is not only related to the number of objects, which can be considered as information *chunks*, but also by their associated complexity [110]. WM capacity is different among individuals, and depends on factors such as age and health conditions [111–114]. The WM capacity dependence on both the environment and on the individual can be represented by the parameters of the SHC based model and, in particular, by the level of the network inhibition (see Fig. 2.6 and [48,50]). Thus, a network can reliably keep the sequence of items based on the WLC principle that supports their right order [115].

2.8. Heteroclinic hubs. semantic memory retrieval and semantic control of information dynamics

We have to distinguish here between hubs in a brain neural network and hubs in the phase space of a functional cognitive network. A brain hub can be identified as having connections to many network nodes (member of a rich club) and

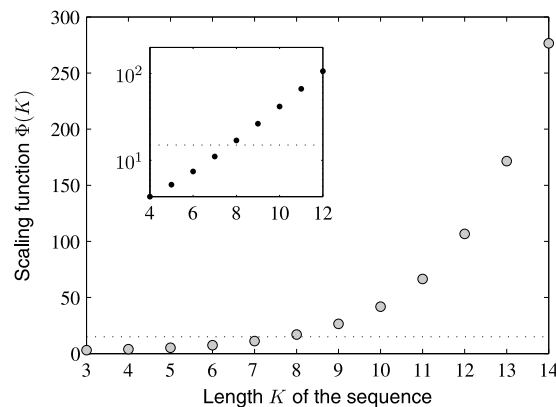


Fig. 2.6. Dependence of sequential memory capacity on the normalized level of network inhibition $\Phi(K) \sim \rho_{ij}/\rho_{ii}$. Inlet: semi-logarithmic coordinates, exponential border of stability with respect to interchange of elements order.
Source: Adapted with permission from [48].

is responsible for the cooperation of different specialized networks [116]. A cognitive heteroclinic hub is a metastable state with a high-dimensional unstable manifold that consolidates different elementary heteroclinic networks in the cognitive phase space.

Semantic hubs play a special role in semantic memory, which stores in a structured manner the facts, concepts and meanings of verbal and nonverbal stimuli. Semantic brain networks encode such information and give form to coherent concepts. Experimental work has provided evidence that physically the semantic hubs are located in the anterior temporal lobe [117]. Rephrasing the famous term “thought as pattern” [118], we can say that the processes of thinking and generation of thoughts are cases of the sequential dynamics of patterns. “Thoughts are not static . . . they emerge and disappear as patterns in a constantly shifting dynamic system” [119]. “Thought arises as a low-dimensional, coherent pattern in an extremely high-dimensional system called the human being coupled to its world” [120].

The control of semantic cognition requires the dynamical interaction between executive and semantic control and the representation systems [121,122]. The semantic control performed by the prefrontal cortex modulates the dynamical representation system that consists of the anterior temporal lobe and modality-specific regions that represent different aspects of the semantic memory.

In particular, in social networks like jazz bands, different moduli are represented by different musicians. A characteristic time course of the jazz improvisation in the band includes alternating segments when a musician is playing solo while the rest of the band is giving him/her rhythmical support. This playing relies in many aspects on the feedback from the body motion and the perception–action cycle [20]. After a while, another soloist comes to the foreground, whereas the former one joins the rhythmic group, and so on. In contrast to the classical chamber music, neither the order of the solo instruments nor the precise duration of the solo segments is rigidly prescribed; instead, dynamics is governed by spontaneous feelings of the musicians. Heteroclinic hub networks can represent such collaboration, where, parameterized by instantaneous contributions of different musicians, the jazz band evolves along a heteroclinic channel. Collaboration is controlled by attention hub networks and the audio–visual interaction between the participants, see Fig. 2.7. Norgaard and coauthors [123] hypothesized that musicians flexibly focus attention during improvisation by storing the binding (concatenating) auditory and motor spatiotemporal patterns in the semantic memory. Interaction of soloists through semantic memory patterns, in fact, is a synchronization process in teacher–learner dynamical systems (see, for example, [124]).

Depending on the strength of the coupling between modules, both results are possible: mutual synchronization or chaotization [91]. In this regard, an insightful discussion regarding the role of conceptual knowledge and cognitive control in a model of social semantics is presented in [125]. In Section 4.4, we will discuss a model of cognitive interaction among multiple musicians in the context of jazz improvisation.

3. Towards a dynamical theory of cognitive activity

3.1. Instabilities, divergence, convergence and attractors — global aspects.

The philosophical and psychological dimensions of consciousness have roots, especially related to linguistics, which can be traced back to the beginning of the previous century. In the 40s, Kenneth Craik wrote “The Nature of Explanation” [127]. In his book, he first laid the foundation for the concept of mentality by stating that the mind forms models of reality and uses them to predict similar events in the future. Craik was one of the pioneers of cognitive science, and his work

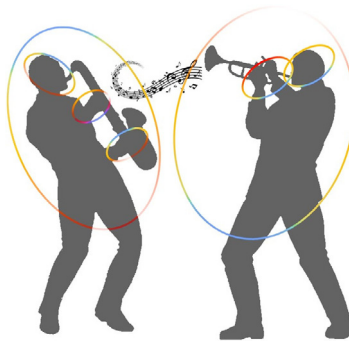


Fig. 2.7. Representation of the audio–visual–musical and motor interactions in jazz playing under attentional control. Collaborative action of a set of semantic memory moduli that represent the activities of different soloists in the band is achieved by sequential attention switching between them. Heteroclinic networks describe such type of dynamics.

Source: Adapted from [126].

influenced many papers about mental modeling (see e.g. [128]) and also the creation of the psychological theories of reasoning and mental logic [129,130].

About half a century later, the cognitive scientist Bernard Baars published his seminal work “A cognitive theory of consciousness” [131]. The main idea of this theory is very attractive: consciousness has an integrative function realized through a brain Global Workspace. This makes sense in a brain that is viewed as an integrated set of highly specialized functional networks. The Global Workspace Theory (GWT) has been generalized by Dehaene and coauthors [132]. A number of modern studies support the assumptions of GWT by showing the reorganization of the whole-brain functional network during cognitive task performance (e.g. see [133]). These modern results showed that when the cognitive demands were raised, the network modularity decreased, and from this change the behavioral performance was predicted. As the task turned more demanding, the number of connector hubs grew, whereas the number of provincial hubs decreased. Neurophysiologists and physiologists are building a detailed map of the global neuronal workspace and addressing the role of many brain regions in a neuronal workspace model of consciousness. In fact, most current theories of consciousness involve distributed large-scale networks throughout the brain. The main ideas behind GWT [134], integrated information theory [135], or global theories of brain neuroenergetics [136] have in common the view that consciousness depends on widespread bilateral brain activity. Just recently, a new dimension of consciousness has emerged: scientists, including mathematicians, have become interested in the dynamical features and modeling of mental processes. An attempt to endow the information integration theory with the abstract mathematical formalism, to introduce for it the space–time continuous description and to develop a measure for the level of integrated information of the system, has been undertaken in [137].

About a century ago, the Russian scientist Lev Vygotsky (1925) provided a definition of consciousness as an active process responsible for the organization of cognitive human functions [138]. Since that time, several authors have proposed different approaches to the understanding of consciousness, creativity and their interaction with autobiographic memory, see e.g. [139–141]. In these views, consciousness deals with intrinsically generated mental processes integrated with perception. Building a dynamical theory of consciousness and cognition, in general, requires three parts:

- (i) It is necessary to use all available information about the functional architecture of such key global brain networks as episodic and semantic memory, working memory, default and attention networks, and their interaction through brain hubs [35].
- (ii) To formulate a mathematical model that represents sequential transient dynamics of such hierarchical brain network. The model has to be:
 - (a) invariant with respect to the informational content of the performance cognitive function,
 - (b) robust against small noise, and reproducible under variations of initial conditions,
 - (c) sensitive to external or internal informational signals, and
 - (d) reflect the fact that any informational processing in the brain is characterized by finite capacity: finite number of units (patterns), these patterns have their own time intervals that in fact can be described as an additional cognitive modality.
- (iii) To analyze hierarchical heteroclinic functional networks in the invariant model in order to find and characterize the hubs that unify memory, attention, default and other heteroclinic networks (see Table 1).

3.2. Sequences of events: time intervals, working memory and conscious dynamics

The core idea about the organization of events and timing of consciousness is related to the variability of sequential organization. The point is on the consciously perceived, temporally extended phenomena (such as the robustness of

the processes and successions of events). Sequential temporal discreteness of cognitive dynamics is responsible for the competitive balance between flexibility and stability, which is a key mechanism for goal-dependent thought generation and creativity. Recent imaging experiments have revealed how specific human brain areas construct networks responsible for the memory of time. In particular, the hippocampus plays a central role in representing temporal contexts of arbitrary lists of sequences of items, or meaningful, lifelike events. In general, the hippocampus, the posterior medial network, and other regions, e.g. the prefrontal cortex, appear to play complementary roles in the formation of temporal information for the memory [142]. In [143] the authors support the hypothesis that time intervals allocate a working memory resource which varies with the amount of temporal information in a sequence. It is reasonable to view interval timing sequence as an additional modality of sequential memory and to model it by a binding network. The generalization of a striatal beat model of interval timing shows how memories for multiple time intervals are represented by neural dynamics and can also be used to explain the mechanisms of resource allocation in the working memory [144].

4. Dynamics of creativity

Creativity is just connecting things. When you ask creative people how they did something, they feel a little guilty because they didn't really do it, they just saw something. It seemed obvious to them after a while. That's because they were able to connect experiences they've had and synthesize new things.

Steve Jobs

4.1. Dynamic metaphors as creativity modalities

Here we consider *jumping out of the line*, i.e., the emergence of metaphors — new dynamics. What do we need for a simple mathematical model of creativity? The goal is to find the bifurcations that correspond to sudden changes of the information processes, caused by self-generation of new ideas and metaphors. It is reasonable to suggest that these transformations of dynamical systems are related to a sudden activation of new degrees of freedom. This can be a way for the generation of metaphors.

In general, the dynamics of creativity results from the interaction of three cognitive modalities. These are: (i) the brainstorming and daydreaming modality (the default network activity is related to it), (ii) the executive autobiographic memory network (which gets activated when a person needs to focus attention on familiar goals) and (iii) the salience network: detection of environmental stimuli and switching of attention between the executive and the default brain networks [145].

Semantic memory allows the inclusion of metaphors into the processes of creativity. A metaphor is a temporal modality in the process of creativity binding. We call a process of connection in time between two unrelated or indirectly linked things – modalities – a *dynamical metaphor*. Existence of such metaphors is a key feature of the creativity process. It can create strong images that can be used with great effect in thinking and in everyday communications, and thus exert strong influence on many consciousness processes. As a rule, the final informational metaphorical image looks absolutely unusual and attracts attention (recall, for example, the graphics of Maurits Escher, or a canvas of Jackson Pollock). A dynamical metaphor is a new non-ordinary connection between previously unrelated modalities. This is critically important for creativity and requires attentional control. The flexible nature of the processes of metaphorical expression and metaphorical thinking is extensively discussed in [146].

Across a wide range of creative or artistic tasks, from composing poetry to music improvisation and sketching pictures, fMRI studies have mapped the brain activity. For example, in [147] it is suggested that the medial temporal lobe may be central to the generation of novel ideas, and that the creative evaluation may extend beyond deliberate analytical processes supported by executive brain regions to include more spontaneous evaluative processes, supported by default and limbic regions. In this way, creative thinking appears to recruit a unique configuration of neural processes that typically do not collaborate [145].

What are the biggest challenges of studying musical or poetic creativity from the dynamic neuroscience perspective? It is not just music or poetry: being an elusive thing, the artistic creativity in general is one of the toughest topics to study and to model. Creativity extends over of a very wide range of human activities, that often cannot be easily brought together. By their very nature, art and creativity are concepts that lack real predictability. Thus, it is not really a natural fit to try to confine them to the constraints of a scientific experiment. Highly creative people are marked by flexible and variable network sequential dynamics. We believe that it is possible to formulate common dynamical principles for the generation of unexpected information. We provide here some examples of structured metaphorical sequences.

Shakespeare, a doubtless genius of metaphors, invented a special one right for our subject: “*memory, the warder of the brain*” (Macbeth). A bright metaphor is usually a non-logical and non-expected leap from the original sequence to another one, possibly with the following return. Two decades ago, Clevenger and Edwards in [148] suggested to use the distance in the semantic space for the quantitative description of metaphors, and for the encoding of metaphors in expressive communication. Metaphor, in fact, is a kind of integrating tool for multidimensional (comprising several

modalities) information through communication between its various levels which is very typical (together with *chunking*: separation into shorter information blocks) for poetry, lyrical music, songs and jazz language as well.

In general, the musical and poetic languages of the last century are characterized by: (1) associativity, (2) multiplicity (dimensionality), (3) codedness and (4) ambiguity of every pattern that is based on dynamic metaphors. We give here just one citation about jazz performance, in order to represent a background against which our dynamical hypotheses would look more fascinating. The famous jazzman Fred Hersch visualizes by words what he plays as “*a kind of big playground with things jumping around on it, usually in terms of melodic movement: things going up this way, balanced by something going down that way*”. He saw “*large masses of things moving along: one string of notes jumping up and down, stopping and twitching around. Music has a feeling of space around it; exist in space, these little mobiles of things. I like to think of music visually like that*”. The description of this dynamical process strongly reminds, as a metaphoric poetic text, the behavior of trajectories in the phase space of a low-dimensional dynamical system with complex behavior, in particular, the emergence of a strange attractor. Semantic creativity involves flexibility and originality resulting for example in irony and metaphors [149,150], where an association between the notions that are either distantly connected or seem completely unrelated, may result in a meaningful linguistic construction. Hence, the semantic network of semantically creative persons may be different than that of less creative people, allowing for more flexible and novel conceptual combinations during semantic processing (see [151]).

The ability of creative individuals to shift their mental focus and switch between different modes of thinking is well documented [152]. These shifts and switches require dynamical interactions among functionally different brain networks, like (in the case of verbal creativity) the interaction between the cerebellum and the task control network. Sun and coauthors also revealed in [152] a close relationship between verbal creativity and high variability of cortical networks involved in spontaneous thought, attention and cognitive control. Psychiatrists know that when their patients imagine future events, the amount of episodic details depends on the kind of initial induction: a control induction without focus on episodic retrieval produces less details than the episodic-specificity induction. Madore et al. have shown that an episodic-specificity induction enhances divergent creative thinking [153,154].

A theoretical approach to describe a metaphor is to relate it – no surprise in our context – to heteroclinic trajectories of the cognitive dynamics. Consider a particular informational item, modeled by the saddle state of equilibrium with a high-dimensional unstable manifold (here “high” may start already with 2). In what direction will the mind wander from this item? Since the disturbances along the eigenvector which corresponds to the largest positive eigenvalue of the Jacobian matrix grow at the fastest, the majority of orbits in the heteroclinic channel leaves the equilibrium along this “leading” direction in the phase space, and, hence, aims at the same particular next item (another metastable state of equilibrium). This does not imply that heteroclinic pathways to other items/equilibria do not exist. However, transitions to other items will be very rare. The arborescent network of heteroclinic connections among various items allows for a huge amount of possible metaphors, but, under usual circumstances, only a minute proportion of these connections gets activated.

We see here, at least, two possible mechanisms for the generation of metaphors. The local mechanism is restricted to the neighborhood of just one equilibrium; for it, the local change of the properties of the phase flow suffices. The action of inspiration (e.g. biochemically manifested through the release of the appropriate agents) could be reflected in changes of the features of the metastable states. In particular, the formerly leading direction may lose its domination, whereas one of the previously non-generic (albeit existent) pathways may overtake its role: as long as the state of inspiration is maintained, the heteroclinic channel directs the mind to a different item. For a sequence of metaphors of this kind, no new heteroclinic trajectories should be created in the functional cognitive phase space; merely some of the existing non-used ones should be activated, at a relatively low energetic cost. A much more violent (and probably a much less common) global procedure would indeed pave new, hitherto non-existent, ways across the phase space, changing its structure and interconnecting the distant items; compared to the local action, this would require larger energetic resources.

It looks that the big poets intuitively master the art of shattering the generic system of connections between the items in the semantic memory, in order to generate surprising associations. We can start right with Shakespeare and recall the famous metaphor occurring to Romeo when, in the moonlight, Juliet enters the balcony. Instead of the worn out analogy between the female beauty and the moon, the metaphor takes a different turn:

“It is the east, and Juliet is the sun.

Arise, fair sun, and kill the envious moon,

Who is already sick and pale with grief”.

This goes far beyond the expected standard comparison: Juliet’s beauty outshines the moon. Or, as another example, we take the famous verse “*Il pleure dans mon coeur*” of Paul Verlaine where the poet looks at his rainy town. Of course, a big city can be compared to anything from an atom to a galaxy, but the most natural, “generic” connection would probably relate a geographical item to another geographical item. In a non-generic poetic way, Verlaine discards the obvious geographical relation (where the rainy Paris might be juxtaposed by e.g. the rainy London — see “The Tale of Two Cities” by Charles Dickens) in favor of the different category: rainy Paris is linked to the rain in the weeping heart of the poet.

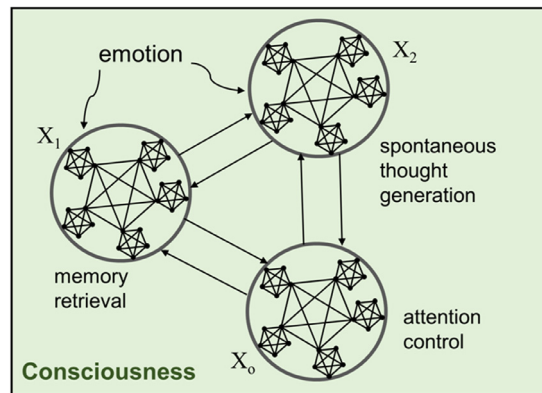


Fig. 4.1. Illustration of the inhibitory interaction between the elementary oscillatory modalities that combine cognitive resources like attention, memory and emotion. In the model equations, such modalities are described as variables X_0 , X_1 and X_2 . All edges correspond to inhibitory connections.

4.2. Resonance interaction of oscillatory creativity modalities

Beatty and coauthors recently summarized experimental works examining cognitive mechanisms of network interactions and correlational studies assessing network dynamics associated with individual creative abilities [155]. There are three cognitive processes related to network interactions during creative performance: goal-directed memory retrieval, cognitive inhibition (see Section 1.3), and internally-focused attention. Studies using prediction modeling indicate that functional connectivity among networks, as recorded in experimental works – particularly the executive control and default networks – can reliably predict an individual's creative thinking ability.

We emphasize that just the knowledge about functional connections of different networks in the brain is insufficient for the description of the consciousness performance and its dependence on environment through perception. For this, it is necessary to understand the dynamical activity of brain networks under the action of external signals: we have to model the non-autonomous cognitive dynamics. In the design of a reasonable dynamical model of creative activity, the following conditions should be fulfilled:

- (i) each modality ought to possess the number of degrees of freedom that allows it to perform its own functions, like working memory retrieval and regenerate the corresponding information items by using its own oscillations without synchronization (1:1 synchronization of all three modalities restricts the diversity of functional activities);
- (ii) phase coordination between the modalities must be the result of nonlinear resonance processes like combination interaction satisfying the resonance relationship between different brain rhythms (see below).

The available evidence from cognitive and neuroscience research reveals, as we discussed, that several creative cognitive modalities, including constructive memory processes, build novel representations, internally directed by attention to support active imagination. This evidence also confirms the relevance of executive control for the implementation of the goal-directed memory, like episodic memory (hippocampal theta-oscillations, 4–8 Hz [156]) and attention (alpha-rhythm, 8–12 Hz [157]). Working together in rhythm coordination with default networks, anxious thinking and active concentration (beta-rhythm, 12–25 Hz [158]), these processes contribute to creative cognition. Event-related alpha-band has been demonstrated over the frontal and posterior cortical sites during ideation associated to divergent thinking [159,160]. It is reasonable to use this information when building a dynamical model for the generation of metaphors.

The rhythm bands of each modality are determined by its own network architecture and intrinsic dynamics. We consider here a modality network hierarchy with two levels and assume that the basic rhythm of each modality is the result of inhibitory interaction between the elementary networks (see Fig. 4.1):

Each of these modalities is the result of the activity of different brain networks that interact with each other. In the functional cognitive phase space, this interaction is represented by a hierarchical heteroclinic network. A chaotic attractor can emerge in the vicinity of the network in a creativity context. The low level of the hierarchy (inside the circles in the representation of Fig. 4.1) features a number of elementary heteroclinic networks (*motifs*) that describe the dynamics of interaction between the basic information items. In different cases, this information may have different sense: a stanza of a poem, an elementary block of a jazz melody, an elementary dance in choreography, etc.

As already mentioned, the ability of creative persons to move their mental focus and change between different thinking modes relates creativity to the dynamical interactions between the brain networks. As a way to estimate the level of creativity, a quantitative measure of semantic distance has been suggested (see [161]). Similar studies complement standard subjective measures and provide objective measures of the creative output. They also allow a more direct way of addressing the role of semantic memory in creativity.

In order to generate novel unexpected associations, cognition should be able to shuffle and combine the notions that are stored in the memory. Here, the role of memory is twofold, both innovative and conservative: it not only supplies information for the creative thought, but also biases the search process in the direction of the already known uses [162], so that a certain cognitive control becomes necessary “in order to inhibit pre-potent response tendencies” [163].

In [141,164] the authors discuss the results of fMRI studies that should clarify which brain processes contribute to the birth of new ideas. The measurements of brain activation and post-scan analyses deal with three kinds of tasks: generation of new original uses of objects, recall of original object uses, and recall of common object uses. New and original ideas generate activation patterns, reminiscent of the recall of the old ideas: activation of bilateral parahippocampal regions. This leads to the conjecture that the “construction of new ideas builds on similar processes like the reconstruction of original ideas from episodic memory” [141]. Compared to the recall, generation of new objects requires higher activation of a focused cluster in the left supramarginal gyrus.

The findings that we have today at our disposal suggest that cognitive processes combine two basic modes of thinking: the automatic heuristic mode and the explicit thinking one. According to [165], the former is used for the information that stays largely invariant over noticeable time intervals, whereas the latter one processes the information that is distinctly different from the previously learned patterns. Studies by fMRI methods in [165] on the effect of the serial order of ideas on brain response disclosed the networks that support the explicit mode of thinking and serve for the transformation of conventional cognitive patterns into new ones. These studies also confirm the relevance of the ability of the working memory to rapidly shift between the thinking modes.

The study of metaphor production offers a new approach to the understanding on how humans generate new ideas. Several neuroimaging studies have used different approaches to investigate the brain regions involved in different types of creative cognition, such as insight problem solving, creative idea generation (i.e., divergent thinking), story generation, and visual problem solving (e.g. see [159,166–168]). Studies focusing on divergent thinking usually involve the protocol of asking individuals to generate novel responses to open-ended problems.

4.3. Chaotic transient dynamics in the Rabinovich–Fabrikant system

Poets, painters, and musicians have long known and reported that creativity unfolds when they are partaking in some kind of chaos. Although they definitely interpret the term “chaos” in its vernacular sense, high sensitivity and unpredictability of this state imply that its adequate mathematical modeling should bring about the kind of dynamics, known nowadays as deterministic chaos. As a dynamical phenomenon, chaos is represented in the phase space of the corresponding dissipative dynamical system by the bounded set of trajectories, characterized by sensitive dependence on the initial conditions: if several close initial conditions are chosen on the neighboring trajectories from this set, the distances between them grow as exponential functions of time. This sensitivity precludes long-time forecasts for individual trajectories. In the case when this set attracts all neighborhood trajectories, we deal with a *chaotic attractor*. Its birth from the simpler forms of dynamics, as a result of the variation of the control parameters, can often be predicted from the analysis of the bifurcation sequences. It is natural to name this kind of chaos predictable or expected. In our opinion, such chaos can represent only some features of creative performance.

Consider a two-level hierarchical heteroclinic network (see Fig. 4.1). As one can see below, we analyze here the dynamics of the slow – envelope – variables X_0 , X_1 , X_2 that describe the creativity process on the upper level of the network hierarchy. The equations for these variables can be derived, if we assume the complete synchronization of the motif heteroclinic oscillations inside all specialized networks: the memory retrieval network, the default mode network (spontaneous thoughts generation) and the attention correction network.

In the shown hierarchy, two heteroclinic triangles or elementary motifs, X_1 and X_2 , represent the activities of different cognitive modalities: (i) episodic (and semantic) memory retrieval and (ii) spontaneous thought dynamics (evident response inhibition). The network X_0 represents the attention dynamics that controls the excitation of the modalities $X_{1,2}$. In general, this network is a non-autonomous dynamical system with three information inputs: e.g. a jazz melody $M_0(t)$ that is the subject or carrier for the processing, i.e., modulation by the heteroclinic network, the emotional events $E_1(t)$ that are remembered better than the neutral events [169], and the emotion control $E_2(t)$ of the generation of thoughts [170,171].

We suppose that the autonomous dynamics of each motif – the heteroclinic triangle – is represented by a limit cycle on the edge of stability, see Fig. 4.2. If $X_{1,2}$ and X_0 are close to the saddle-focus steady states, these modalities oscillate in a nearly harmonic fashion (panel A in Fig. 4.2):

$$X_j(t) \approx A_j(t) e^{i(\omega_j t + \varphi_j)} \quad (4.1)$$

with (some) frequencies ω_0 , ω_1 , and ω_2 . Since the saddle-focus is a collective state of equilibrium, small-scale oscillations around it are collective as well: at every instant of time, amplitudes of all participating modes are of comparable order, all of them simultaneously present in the dynamics.

However, if such cycle is close to the heteroclinic contour, the motif oscillations will be strongly nonisochronous: long epochs of slow motion across the neighborhoods of the saddle equilibria in the “corners” alternate with relatively fast motions along the segments that connect the equilibria (panel B in Fig. 4.2). In contrast to collective oscillations from the

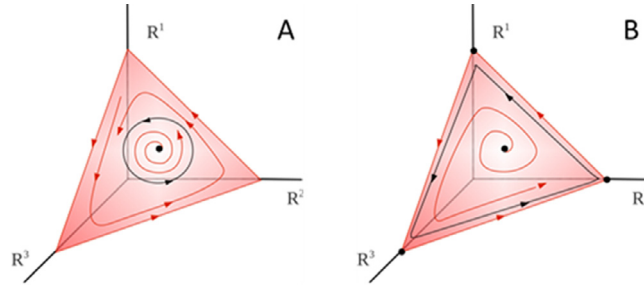


Fig. 4.2. Rhythmic activity shown by a simple motif is strongly dependent on the parameters: the panel A shows quasi-harmonic oscillations, whereas in the panel B the limit cycle is close to the heteroclinic contour, therefore the oscillations are strongly nonisochronous.

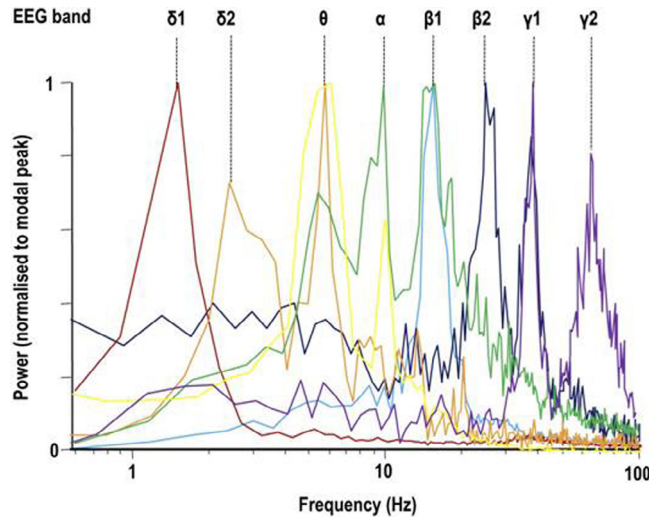


Fig. 4.3. Multiple modal peak frequencies of persistent rhythms generated in local circuits of the neocortex in vitro. Source: Adapted with permission from [172].

panel A, here the dynamics is sequential: during alternating long passages close to saddle equilibria, typically just one mode features significant amplitude whereas the others are subjected to strong damping.

Whenever the autonomous oscillations of $X_{1,2}$ weaken, the oscillation of X_0 supports them with energy in the process of the nonlinear interaction. Such energy exchange between the three modalities has to be mutual. This happens close to the resonance (the so-called modulation instability):

$$2\omega_0 = \omega_1 + \omega_2 + \Delta_\omega \quad (4.2)$$

where Δ_ω denotes the detuning from the exact condition of resonance for three frequencies.

This mechanism can explain the coordination of different brain rhythms. Experiments in vitro have shown an important phenomenon in this context: the autonomous part of the neocortex is able to generate a wide spectrum of coordinated brain rhythms (see Fig. 4.3).

The complex Ginzburg–Landau (CGL) equation has been used for modeling cortical dynamics including neuronal avalanches and their relationship to brain rhythms [173–175]. Let us consider, in the framework of the CGL model, the description of three inhibitory oscillatory cognitive modalities:

$$\begin{aligned} \dot{X}_0 &= 2\sigma X_1 X_2 X_0^* e^{-i\Delta_\omega t} + \gamma X_0 + i\alpha X_0(|X_0|^2 + 2|X_1|^2 + 2|X_2|^2) \\ \dot{X}_{1,2} &= \sigma X_{2,1}^* X_0^2 e^{i\Delta_\omega t} - \nu_{1,2} X_{1,2} + i\alpha X_{1,2}(2|X_0|^2 + |X_{1,2}|^2 + 2|X_{2,1}|^2) \end{aligned} \quad (4.3)$$

Next, for simplicity, we suppose that the damping coefficients $\nu_{1,2}$ coincide: $\nu_{1,2} = \nu$. In this case,

$$\frac{d}{dt} (|X_1|^2 - |X_2|^2) = -2\nu (|X_1|^2 - |X_2|^2) \quad (4.4)$$

so that the intensity of the dissipative modes becomes, in the course of time, practically equal. Then, after the appropriate rescaling (see [176] for details) we obtain for the new rescaled variables $x(t)$, $y(t)$ and $z(t)$ the three-dimensional

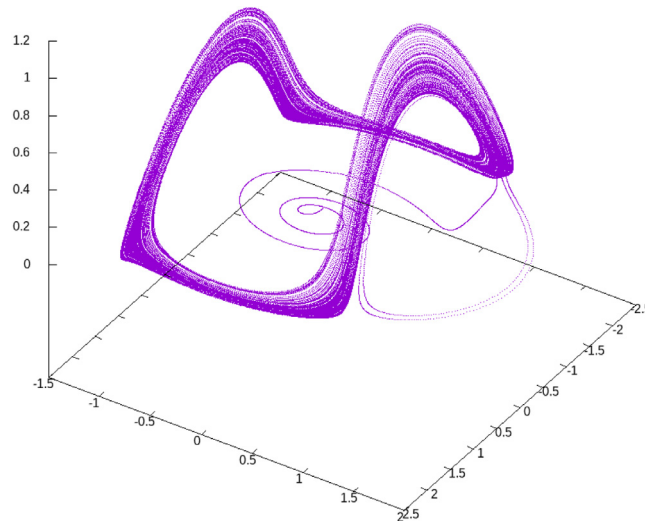


Fig. 4.4. Rabinovich–Fabrikant attractor at parameter values $\gamma = 0.1$ and $\nu = 0.2715$.

Rabinovich–Fabrikant (RF) model:

$$\begin{aligned}\dot{x} &= y(z - 1 + x^2) + \gamma x \\ \dot{y} &= x(3z + 1 - x^2) + \gamma y \\ \dot{z} &= -2z(\nu + xy)\end{aligned}\tag{4.5}$$

where

$$\begin{aligned}x &= \left(4 \left| \frac{\alpha}{\Delta\omega} X_0 \right|^2\right)^{1/2} \cos \Phi \\ y &= \left(4 \left| \frac{\alpha}{\Delta\omega} X_0 \right|^2\right)^{1/2} \sin \Phi \\ z &= 2 \left| \frac{\alpha}{\Delta\omega} X_1 \right|^2 \\ 2\Phi &= \left(\Delta\omega t + 2 \arg \frac{X_0}{X_1} \right) \text{sign } \alpha\end{aligned}\tag{4.6}$$

There are different mathematical images of creativity, depending on the character of the creativity goal. It is reasonable to separate at least two types of creativity: (i) the emergence of the process, i.e. a new type of dynamics that is characterized by infinitely long duration and can be recalled as a process in time: music, poetry, dance etc.; and (ii) the emergence through a new complex transient sequential process of an unusual informational pattern like the already mentioned examples of Pollock canvases and Escher architecture images, or unusual culinary dishes. An example of image of the first type of creativity is presented in Fig. 4.4: this is a strange attractor. The image of the second kind of creativity is a new stable informational pattern, i.e., a stable steady state. The complexity of such informational pattern depends on the length of the transient sequence that ends at the corresponding steady state (see below Eqs. (4.9)–(4.10), and [177]). The RF model has demonstrated creativity images of both types (see Fig. 4.5).

The set of Eqs. (4.5) illustrates the emergence of chaotic sets in a system with resonance interaction of the modes [176, 178]. The onset of chaos in this model, as a result of the parameter variation, is difficult to predict because of multistability and transient chaoticity phenomena in the phase space. In the context of creativity, it should be mentioned that when the signal, produced by the orbit wandering on this particular chaotic attractor, is mapped onto the sounds of drums and percussion, a unique “fractal” music is generated (<https://www.youtube.com/watch?v=kh6ZLvpWr5k>). This system, with its third-order nonlinearities, displays “virtual saddles” in addition to chaotic sets with different shapes, hidden chaotic attractors and hidden transient chaotic orbits [178–180], see Fig. 4.4.

Like many other complex dynamical systems, the models of cognitive activity often feature multistability (see e.g. [181]): coexisting attractors. In such situations, the choice of the final state depends on the initial conditions, and it is commonly highly sensitive towards noise and fluctuations of the system parameters. Recent studies relate multistability to occurrence of unpredictable chaotic transients and so-called “hidden attractors” [179,180]. To understand the global dynamics of such systems one has to identify the possible transient chaotic sets, the attractors and their basins. Numerical

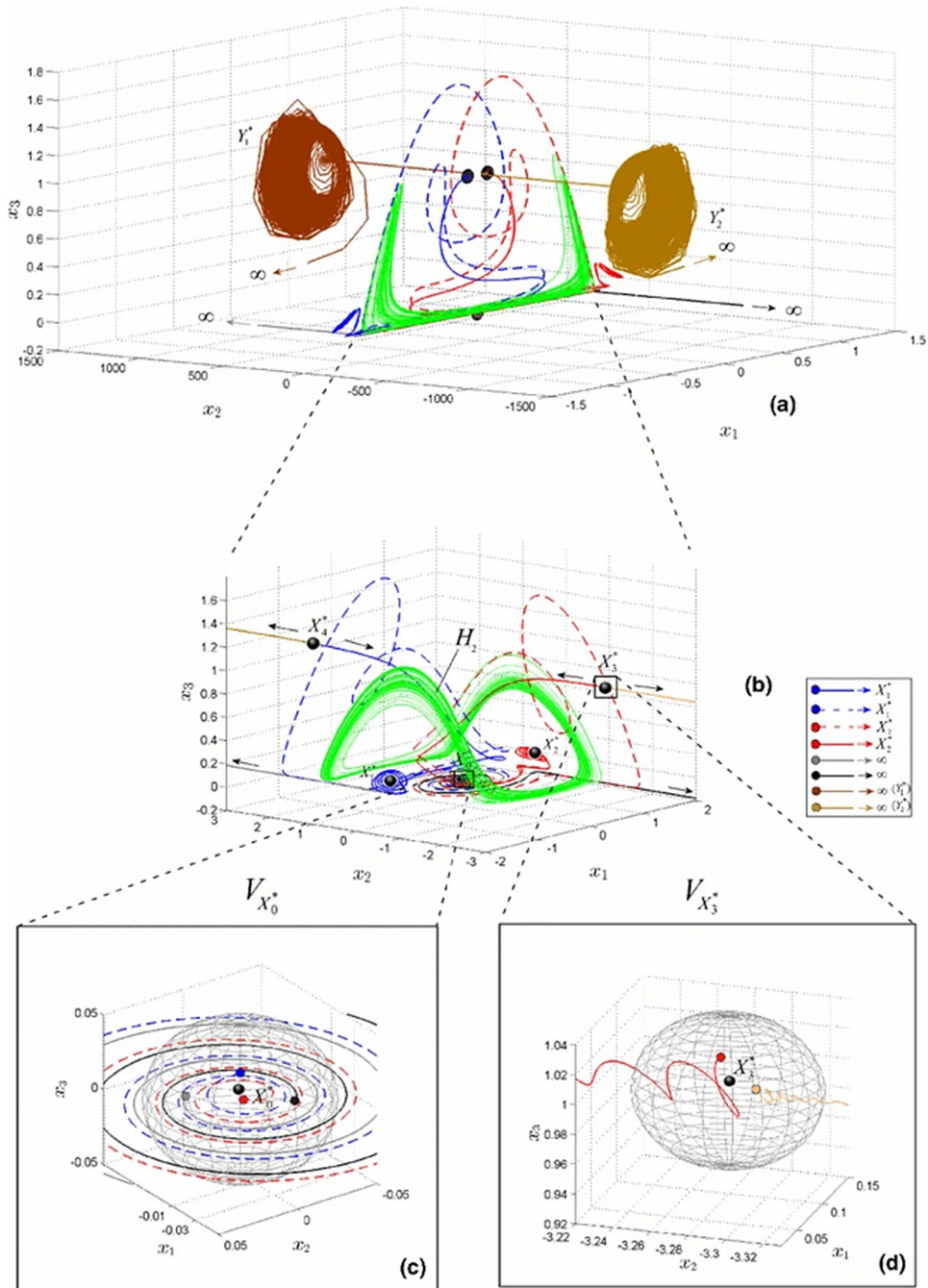


Fig. 4.5. An illustration of a hidden chaotic attractor. (a, b) The hidden attractor is depicted here in green with trajectories diverging to infinity and others attracted by the stable equilibria. (c, d) illustration of other trajectories starting from different vicinities.
Source: Adapted from [179] with the permission of Springer Nature.

localization of hidden attractors is often difficult, since they do not evolve from simple states of equilibrium through transparent bifurcation routes.

4.4. Sequential creativity dynamics and emotion

It is generally accepted that emotion can be defined as responses to events that support and adapt the way we think and behave [182]. Emotions include a number of components like the cognitive appraisal of events, motor expression,

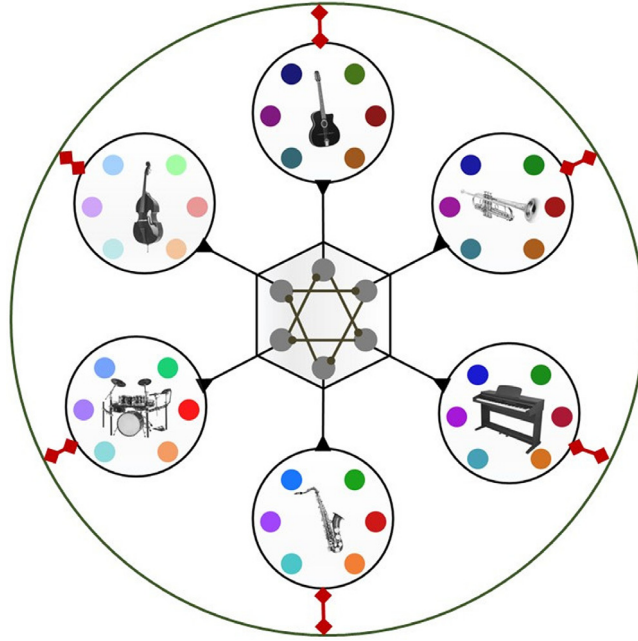


Fig. 4.6. Collaborative creativity of a set of soloists (hexagon network) is achieved by sequential switching between them (circles). In this model the soloists control the sequence of creativity ideas within the initial state demand of other within a group. Such dynamics maximize the creativity of the soloist. The common interaction among soloists uses attention resources and their auditory and visual information from each other (red arrows), and builds their common goal in the creativity process, see also Fig. 2.7.

feeling yourself and others. Positive emotions (joy, satisfaction, pride) augment creativity during idea generation and increase flexibility of conceptual combination, while negative emotions reduce the flexibility of information available for the creative process and the chance for generation of new ideas [183].

Let us come back to the example of jazz soloist collaboration and consider a model of six-soloist activity. The collaborative creativity of the set of soloists is achieved by the sequential switching of their playing in concert (see Fig. 4.6).

The neural substrates that underlie spontaneous musical performance have been analyzed in [184]. Improvisation of professional jazz pianists playing on a specially constructed keyboard was investigated by means of fMRI. The overall spatial pattern of activity during improvisation was found to be incoherent: focal activation of the medial prefrontal cortex coexisted with deactivation of the lateral orbital region as well as of the rear prefrontal region. This decentralized pattern, apparently inherent for the spontaneous improvisation, was interpreted in [184] as development of internally motivated stimulus-independent behavior in the absence of conscious control of the performance.

To illustrate the main point about spontaneous chaotic playing with different ideas, we do not take into account here another aspect of the creativity process: emotion dynamics. Usually all partner jazzmen play up to a soloist. We ignored this complication because we believe that it contributes little to our model of creativity while complicating the corresponding dynamical model. Thus, we omit direct connections between players.

An appropriate simplified model for the description of the creativity variables x_i^m depending on the attention control network can be written on the base of the canonic equation (2.3) in the form

$$\tau_i^m \frac{dx_i^m}{dt} = x_i^m \left[\sigma_i^m(R^m, S^m, C^m) - \sum_{j=1}^{K^m} \rho_{ij}^m x_j^m \right] \quad (4.7)$$

Here τ^m characterizes the timescale of the switching between the ideas, S is the vector of sensory inputs, $\sigma_i^m(\dots) \geq 0$, $\rho_{ij}^m \geq 0$ is the connection matrix between the competitive ideas used by the m th soloist. Results of the numerical simulation of this model are displayed in Figs. 4.7 and 4.8.

Fig. 4.7 presents the time series corresponding to an independent soloist improvisation. For simplicity, four creativity ideas out of six are plotted. Fig. 4.8 corresponds to the same player interacting with the other musicians as a band.

To characterize quantitatively the value of the information generated by the creativity process, we calculated the Lyapunov dimension for each network using the Kaplan–Yorke formula:

$$D_{KY} = j + \sum_{k=1}^j \frac{\lambda_k}{|\lambda_{j+1}|} \quad (4.8)$$

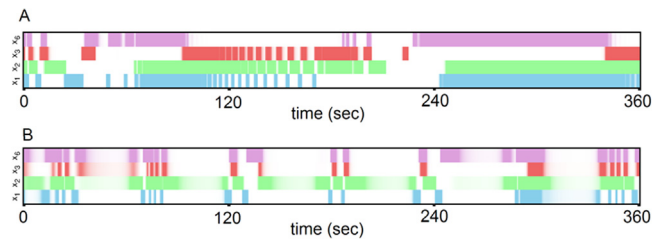


Fig. 4.7. Modeling of individual soloist improvisation by a heteroclinic network. A: a jazzman independently plays ($D_{KY} = 4.05$). B: activity of the same player is sequentially switching with the other players of the band under the attentional network modulation; in this case $D_{KY} = 6.01$. Different colors represent the temporal patterns corresponding to different creativity.
Source: Adapted with permission from [50].

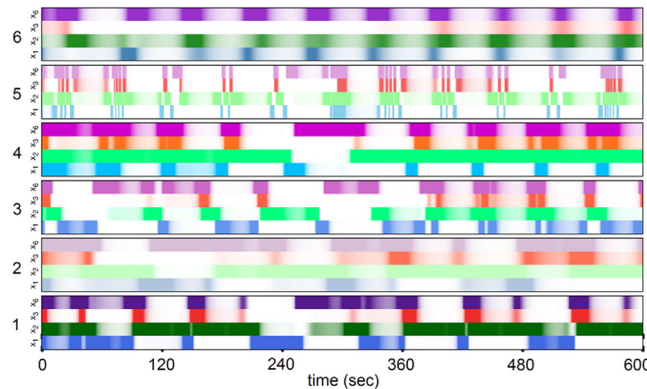


Fig. 4.8. Analysis of creativity in the sequential patterns produced by a model of the interaction between members of the music band sketched in Fig. 4.6. Each member of the band uses his/her own memory about the sequence of the preceding improvisation.
Source: Adapted with permission from [50].

where $\lambda_1 \geq \dots \geq \lambda_n$ are the Lyapunov characteristic exponents, and j is the largest integer for which the sum $\lambda_1 + \dots + \lambda_j$ is positive.

4.5. Surprise and instability

When we hear a good poem and start to think about it, we deal, like in real life, with an unpredictable sequence of cognitive episodes that is distinct from the cognition of non-creative content. Reading or reciting a poem is a sequential dynamical process that at the first instances is unpredictable. Interpretation uncertainty can be characterized by the Kolmogorov–Sinai entropy or information dimension. The application of the formulated above dynamical paradigm to the poetry processes, is, in fact, a development of the well-known idea that the creation of poems is a sequential variation of the semantic constructions around the preliminary theme. In particular, the poet Joseph Brodsky in his Nobel lecture said “One who writes a poem writes it because the language prompts, or simply dictates, the next line. Beginning a poem, the poet as a rule doesn’t know the way it’s going to come out, and at times he is very surprised by the way it turns out, since often it turns out better than he expected, often his thought carries further than he reckoned. And that is the moment when the future of language invades its present”. There are three mental modes: analytical, intuitive, and the mode that was known “to the Biblical prophets” (in our understanding – instabilities), that participate in the dynamical process of the poem creation; these modes have to be bound. As an example of semantic unpredictability we quote here two stanzas from the famous poem of Thomas Eliot:

*I have seen them riding seaward on the waves
Combing the white hair of the waves blown back
When the wind blows the water white and black.

We have lingered in the chambers of the sea
By sea-girls wreathed with seaweed red and brown
Till human voices wake us, and we drown.*

(T. Eliot, 1920, The Love Song of Alfred Prufrock)

What does the flow of the poem mean? One can imagine several different poetically reasonable branches which can nicely continue any line in different ways. This illustrates variability and unpredictability of the poetic process. Especially surprising for the reader is the end of the poem. Is this amenable to mathematical modeling?

The canonical model that we have built above is applicable for the description of cognitive dynamics of poetry creation. The typical solution of that model is a sequence of metastable states that represent new concepts or images arising during the writing through association between the modes of semantic network. This means that, in our model, the process of the poem creation is represented in the phase space of the model by a one-directional chain of metastable states (see also Fig. 1.2A). A metastable state is triggered to the next one by a structural analogy, semantic similarity, polysemy, etc. On some steps, the chain of metastable states becomes unstable and the system jumps to another branch of the main tree, randomly choosing between the new modes of the semantic network. Such uncertainty is inherent for a mathematical representation of the creativity process. In the framework of the model discussed above, we can analyze the influence of the information structuralization in the form of a sequence of chunks on the stability of hierarchical creativity process. Creative cognitive ability is correlated with the dynamic reorganization of resting-state functional brain networks. In this context, big data fMRI analysis has established a relation between creativity and the temporal variability of cortical and cerebellar regions during the resting state. The link between variability and creativity is supported by the hypothesis of randomness in the transition from one network state to another [152].

4.6. Dimension of cognitive information flow

One of the most constructive features of consciousness and creativity is the brain information integration process [185]. It is implemented in the brain by the binding mechanism. Binding is a nonlinear dynamical phenomenon that transiently, i.e., sequentially in space and time, unifies the information carried by the flows of different modalities. We name it the *Consciousness Information Flow* (CIF).

In the functional cognitive phase space the binding dynamics of the flow is represented by phase trajectories disposed in the vicinity of the binding heteroclinic network that is formed by metastable states – the hubs – with multidimensional unstable manifolds. These trajectories can be viewed as elements of many-dimensional CIF. The transient dynamics represented by such channels is robust and reproducible.

To quantitatively characterize the complexity of the CIF, let us introduce a new function that we call the *local Information Flow Dimension*, as

$$D_{\text{IF}}(L) = \sum_j^L \Delta D_{\text{IF}}(I) \quad (4.9)$$

where

$$\Delta D_{\text{IF}}(I) = J_l + \sum_{j=1}^{J_l} \frac{\lambda_j^l}{|\lambda_{J_l+1}^l|} \quad (4.10)$$

Here L is the total number of metastable states (saddles) visited by the system until the time t_L , l is the index of the saddle along the channel, λ_j^l are the ordered ($\lambda_1^l \geq \dots \geq \lambda_n^l$) characteristic Lyapunov exponents at the saddle l , whereas the integer J_l is defined by the conditions:

$$\sum_{j=1}^{J_l} \lambda_j^l > 0, \quad \sum_{j=1}^{J_l+1} \lambda_j^l < 0 \quad (4.11)$$

In the case when the unstable manifolds of all saddles along the heteroclinic channel are one-dimensional,

$$D_{\text{IF}}(L) = \sum_l^L \left(1 + \frac{1}{\nu_l} \right) \quad (4.12)$$

where ν_l is the saddle value (2.7) of the saddle l . In this situation all $J_l = 1$, and the ratio of positive Lyapunov exponents to the negative ones is simply $1/\nu_l$ (recall that for the channel stability the product of all saddle values in the channel has to be larger than 1).

We illustrate the usefulness of the D_{IF} for the quantitative description by the example of heteroclinic binding problem. In the heteroclinic skeleton, sketched in the second row of Table 1, each saddle along the “binding heteroclinic channel” has a two-dimensional unstable manifold. This implies $J_l = 2$ for all l . Thereby, the above estimation of the D_{IF} indicates that the flow capacity for a binding channel is at least twice as large as the D_{IF} of three independent channels. This result can be interpreted in the following way: the information flow capacity characterizes the complexity level of the trajectories within a network of heteroclinic channels. We can hypothesize that such complexity supports fast and versatile mechanisms of encoding the information about a subject. It seems promising to relate the function D_{IF} with the Shannon information and the capacity dimension of chaotic sets, see [186]. We are sure that this will be assessed in the near future.

5. Conclusions and outlook

Here we have considered consciousness as a transient multi-modal spatio-temporal process in the brain. This process is the result of integrated activity of many functionally specialized global brain networks that represent different cognitive modalities like autobiographic memory, attention, thought generation, sensory perception and others. Mathematically such transient dynamics can be represented by a network of heteroclinic trajectories in the functional cognitive phase space. The image of the life of consciousness is a phase portrait of the dynamical model that mimics the activity of the brain.

5.1. Timing in conscious dynamics. multimodality binding, synchronization and entrainment

As we already discussed, consciousness is a robust sequential process; its purpose is to recognize the present and to predict the future. This transient integrating process is based on several cognitive modalities. Among them, the key ones are: the autobiographic memory – including episodic and semantic memories, the attention control, and self-awareness of the generation of thoughts. All these modalities themselves are sequences of events localized in time. The robust retrieval that we discussed on the example of one modality (working memory) means the reproducibility of the order of the items within the sequence. This robustness is possible, provided that the sequence is short enough [48].

It is reasonable to hypothesize that the robustness of a cognitive multimodal sequence is determined by the stability of the binding sequence. The specifics of such problem is related to the fact that one modality is a sequence of time intervals. This modality is often metaphorically named as “ebb and flow” of the tide in our thoughts, or *the consciousness* in general [187]. Since the temporal organization of events into chunks is, in general, non homogeneous, it is necessary to analyze the dynamical control of the time intervals. In the simplest cases it can be mutual synchronization or, even more efficient in the multimodality cases, the entrainment.

Jazz improvisation with variable rhythms, and recital of poems with different cadence are well known examples of memory retrieval with flexible variation of the time intervals between the neighboring cognitive and behavioral events. In contrast to the rhythmic timing of intervals, when the ratio of the chunk duration to the interval between the chunks does not depend on local time [1,188], the description of conscious activity with variable time intervals between the events is more complex and needs new approaches. An interesting example is the occurrence of switching between multimodal coordination patterns and of synchronization between them, which was observed in multimodal coordination behavioral dynamics [189].

5.2. Emotional and attention modulation of sequential interval timing

What we hear in a musical concert seems to be determined by the sound reaching our ears. However it also depends on our memories about the sounds that we have heard before.

In fact, we feel a multimodal mixture of the original arrangement of the music with the current representation. For example, one of such modalities can be jazz improvisation with sequential interval timing. In principle, a listener can control the current temporal performance by emotion and attention that are strongly interconnected with working memory [190]. A similar statement refers to the emotion. The corresponding neuropsychological mechanism is known: experiments have provided evidence that when the amygdala is connected with the hippocampus and the prefrontal cortex, i.e., when they are forming a joint network, together they play a key role in retrieving emotionally experienced events [191,192]. Fayolle and coauthors [193] also showed that the emotion of fear speeds up the internal clock. It is now well understood that biological internal clocks are modulated by attention and arousal. Preliminary adjustment to a rhythmic external signal (*pacing*) is also known to have a role in the setup of timing intervals [194]. A dynamical model can help to address the hypothesis that observing emotional facial expressions distorts subjective time perception through the interaction, e.g., of the neural network responsible for the processing of facial expressions with the brain network involved in timing [195]. In general, the mutual dynamics of such networks can be quasiperiodic or chaotic, see also [52].

5.3. The socio-brain. temporal coordination between brains

Challenging problems about the timing of consciousness dynamics evidently have to be formulated for the case of social cognitive networks. Certain social groups, like the orchestras and the sport teams, engage in activities that demand a high level of time coordination between the participants. Success in some of these activities (classical ensemble music, synchronous swimming) assumes rather tight timing among the group members; in other situations (jazz bands, cycling and rowing teams) an individual, albeit keeping some freedom in her/his pace, remains restricted by the timing of the group as a whole. How is the adjustment implemented within the group?

Understandably, for the success of the group action, the timescales of relevant processes *inside* the brains of the participating individuals should be adjusted to the characteristic times of the interaction *between* the group members. Social modality affects time perception, so that the subjective durations of time intervals become altered, changing in their turn the feeling of being “in” or “out” of synchronization. The authors of [196] suggest that the core of these reciprocal interactions is an “internal clock” involving subcortical orchestrated oscillations that represent temporal information, such

as duration and rhythm, as well as insular projections linking temporal information with internal and external experiences. The timing of social relative to nonsocial stimuli changes the interconnection in corresponding networks. Together, these reciprocal pathways may enable the exchange and respective modulation of temporal and social computations. In our opinion, the analysis of temporal binding dynamics in a social group is a promising direction.

5.4. From neuroscience to robotics, artificial intelligence and biomedical applications

The formalism described in this paper has applications beyond neuroscience research. Classic robotic paradigms typically use rule-based decision that leaves little space for creativity or allowing just programmed creativity. The perspective of seeing the behavior as a succession of robust sequential interactions provides the opportunity to employ the described models in order to drive representation of cognitive interactions into the realm of robotics and artificial intelligence. Previous work has shown that winnerless competition and heteroclinic networks can be used in robotics, robot–human interaction or autonomous artificial agent research [197–201], including also the realm of machine learning [96].

Today we are witnessing a shift in our human-focused social paradigm towards a greater involvement of artificial cognitive agents in our everyday life. The collaborative scenarios between humans and robots become more frequent and will have a deeper influence on our daily routines. The mutual interaction between people and robots is a source of several specific problems related to the quality of continuously changing in time (transient) exchange of information between the collaborators. The main question in this context is: can we trust the robot's information to use it as it is, and vice versa? The on-line decision to this question needs the attentive evaluation of the current situation against the information kept in episodic and semantic memory. Such activity is an example of a conscious robot everyday problem.

Now conscious robotic cognition calls for memory information integration including language and sensor/motor integration. In fact, all our discussion above is based on the idea that consciousness is an integrated dynamics of spatiotemporal information patterns from outside and inside the brain. It looks very natural to use the same modeling approach – sequential heteroclinic dynamics – for the description and development of the *robotic consciousness* in order to organize the collaboration between sentient robots and humans.

A major challenge for such collaboration are the automatic correlation of languages and the denoted sensorimotoric experiences, commonly known as the Semantic Gap problem. The discussed “heteroclinic universality” can be an efficient tool for solving this problem, as it allows hierarchical sequence binding and coordination.

Modern neuroscience provides new evidence on the structure of semantic memory, and points to the fact that semantic information is multisensory, multimodal and distributed. In the dynamical language, it means that the process of binding in the “robotic cognition” has to be multimodal and deeply anchored in time.

The coordination of timing clearly plays an important role in the efficiency of human–robot collaboration processes. The above discussed heteroclinic synchronization phenomenon offers a possible mechanism for joint “human–robot mind” temporal coordination and control.

The description of robust sequences related to creativity neural processes and the identification of the corresponding dynamics and bifurcations can also lead to novel approaches in the context of biomedical research and clinical applications [52,54,202]. The dynamical characterization of resting state and stimulus-induced cognitive neural activity has been suggested for the design novel biomarkers of brain diseases [18,203,204]. Recently, the characterization of sequential brain dynamics has also been proposed for the analysis of techniques that use rhythmic stimulation such as steady state visually evoked potentials and intracranial stimulation in the context of different research and biomedical applications [205]. All these works point out the need and potential wide impact of the theoretical framework and the models discussed in this paper.

As we discussed above, nature uses universal dynamical principles such as transitivity, robustness, and minimization of resources for realization of many cognitive functions: perception, reinforcement memory, decision making, creativity, and consciousness. The understanding and prediction of transient information processes in the human brain and their dynamical images are challenging and attractive for physicists, mathematicians, psychiatrists, neurophysiologists, computer scientists, and medical doctors. Today these scientific issues are also addressed within the context of AI and robotics. Of course, it is hardly possible to address the vast array of problems in the framework of a single review, and the material that we presented here is just the tip of an iceberg. We are confident that such topics as nonlinear dynamics of social human–robots groups, cooperation of the brain with AI in decision making, and the creation of a joint balanced future will be the subject of many new efforts in forthcoming years.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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